Exam 3 Outline (Motivating Questions)

9.8: Arc Length

- How can a definite integral be used to measure the length of a curve in 2- or 3-space?
- Why is arc length useful as a parameter?

12.1: Vector Fields

- What is a vector field?
- What are some familiar contexts in which vector fields arise?
- How do we draw a vector field?
- How do gradients of functions with partial derivatives connect to vector fields?

12.2: The Idea of a Line integral

- What is an oriented curve and how can we represent one algebraically?
- What is the meaning of the line integral of a vector-valued function along a curve and how can we estimate if its value is positive, negative, or zero?
- What are important properties of the line integral of a vector-valued functions along a curve?

12.3: Using Parameterizations to Compute Line Integrals

- How can we use a parametrization of an oriented curve C to calculate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$
- How does the parametrization chosen for an oriented curve C alter the value of the line integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$
- What can be said about the line integral of a vector field along two different oriented curves when the curves have the same starting point and same ending point?

12.5: Path Independence and FTC for Line Integrals

- What characteristic of a vector field $\boldsymbol{F}$ will make $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ have the same value for every oriented curve from a point P to a point Q?
- What special properties do gradient vector fields have?
- Given a gradient vector field $\boldsymbol{F}$, how can we efficiently find a potential function f so that $\boldsymbol{F} = \nabla f$

12.7: The Curl of a Vector Field

- What is meant by rotation of a vector field in a plane?
- How can a two-dimensional measurement of rotation be generalized to work in three dimensions?
- How can the rotational strength of a vector field be measured?

12.8: Green's Theorem

- How can we calculate the circulation of a two-dimensional vector field $\boldsymbol{F}$ around a closed curve when $\boldsymbol{F}$ is not path-independent?

- What is the meaning of the double integral of the circulation density of a smooth two-dimensional vector field on a region R bounded by a closed curve that does not intersect itself?

11.6: Surfaces Defined Parametrically and Surface Area

- What is a parameterization of a surface?
- How do we find the surface area of a parametrically defined surface?

12.9: Flux Integrals

- How can we measure how much of a vector field flows through a surface in space?
- How can we calculate the amount of a vector field that flows through common surfaces, such as the graph of a function $z = f(x, y)$

12.6: The Divergence of a Vector Field

- How can you measure where a vector field is created (or destroyed)?
- How can you measure where a vector field's strength is increasing or decreasing?
- What does the divergence of a vector field measure and how can you visually estimate whether the divergence of a vector field is positive or negative?

Exam 3 Outline (Important Concepts and Formulas)

- Arc-length formula
- Reparametrization and arc-length parameterization
- Vector fields
- How to plot a vector field
- Gradient vector fields
- Line integrals with and without using parameterizations
- Work done by a vector field
- Path-independence
- How to tell if a vector field is path-independent
- FTC for Line Integrals
- Potential functions and how to find them
- How to compute curl of a vector field in 2d, 3d
- Interpretations of curl
- What does it mean if $curl(\boldsymbol{F}) = \mathbf{0}$? What if curl is non-zero?
- What is a closed curve?
- What is a simply connected region?
- What is a simple closed curve?
- What is Green's theorem and when can we use it?
- Circulation & circulation density of a vector field
- Parametrizations of surfaces
- Common examples:
    - Graphs of functions of the form $z = f(x, y)$
    - Surfaces of revolution
    - Cylinders
    - Spheres
    - Planes
    - Cones
- Surface area formula from a parameterization
- What is the normal vector of a surface? How is it computed and how do you visualize it?
- What does it mean for a surface to be oriented?
- Flux (i.e. Surface) integrals
- How to compute surface integrals with(out) using a parameterization
- Divergence of a vector field
- Source-free / Divergence-free
- Interpretations of divergence
- Approximation of flux by divergence and small surfaces