MATH 208, EXAM 1

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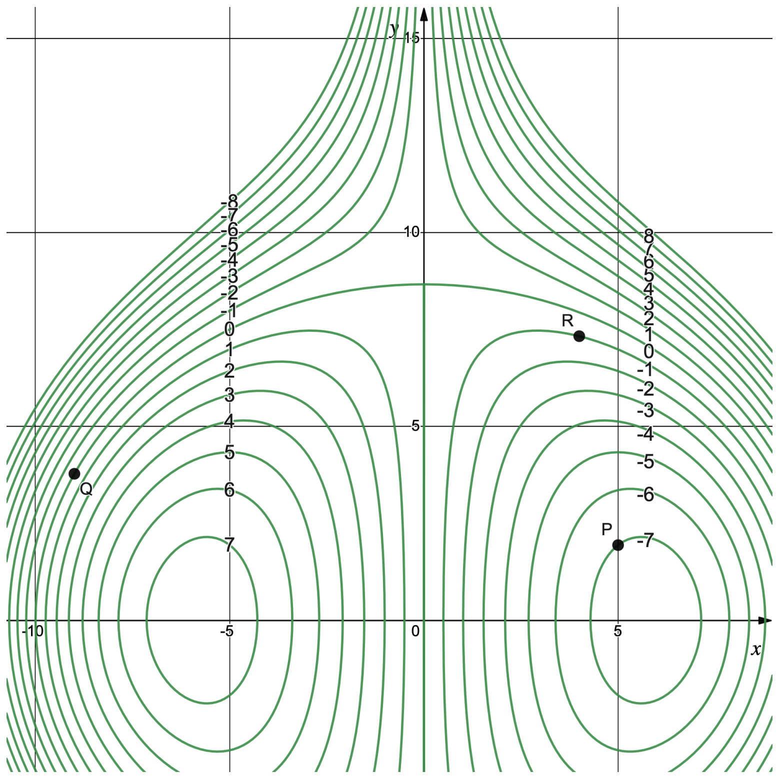
Name: ______________________ NUID: ______________________

Instructions.

- You should have 9 pages on which 7 problems are printed.
- You have 50 minutes: the exam will begin on the hour and end promptly, 50 minutes later.
- Show all work unless otherwise specified. What you write on the page must convince me that you understand the problem and its solution.
- Read each problem carefully.
- You do not need to simplify your answers, unless the instructions for a problem indicate otherwise.
- You are not allowed a calculator, notes, textbooks, or access to any electronic devices.
- Don't panic. Good luck!

Date: Fall 2022.

Below is pictured a contour map of a function $f(x, y)$. (Notice that some labels for contours appear in the left half of the picture, and others appear in the right half.) Problem 1 refers to this graph.



Problem 1 (2 points each)**.** Answer each question about the partial derivatives of f at the point Q.

(a) At the point Q, f_x is …
- ☐ POSITIVE
- ☐ NEGATIVE
- ☐ APPROXIMATELY 0

(b) At the point Q, f_y is …
- ☐ POSITIVE
- ☐ NEGATIVE
- ☐ APPROXIMATELY 0

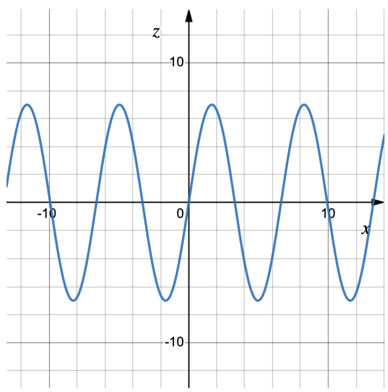
(c) At the point Q, f_{xx} is …
- ☐ POSITIVE
- ☐ NEGATIVE
- ☐ APPROXIMATELY 0

(d) At the point Q, f_{yy} is …
- ☐ POSITIVE
- ☐ NEGATIVE
- ☐ APPROXIMATELY 0

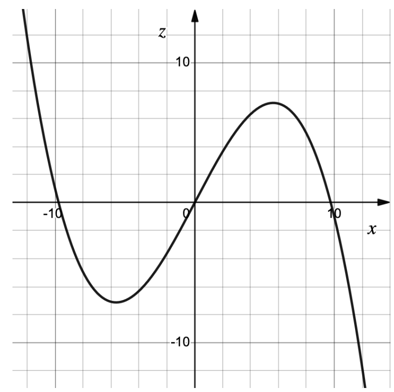
(e) Which of the graphs below is the correct graph of the x-trace $f(x, 1.9)$? (The point P is $(5, 1.9)$.)

- ☐ graph (i)
- ☐ graph (ii)
- ☐ graph (iii)
- ☐ graph (iv)
- ☐ graph (v)

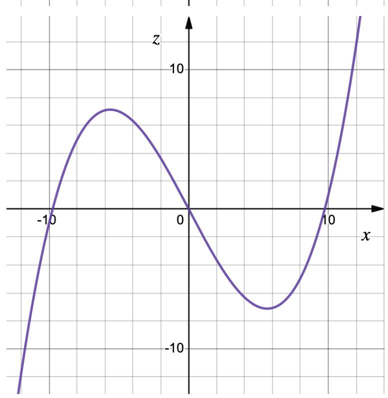
(i)



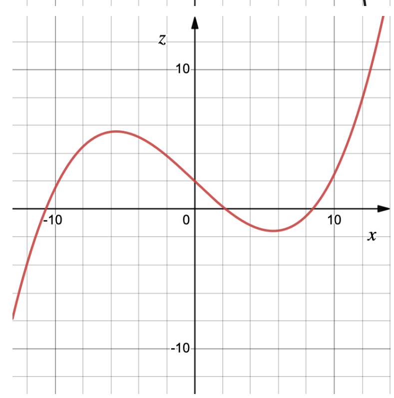
(ii)



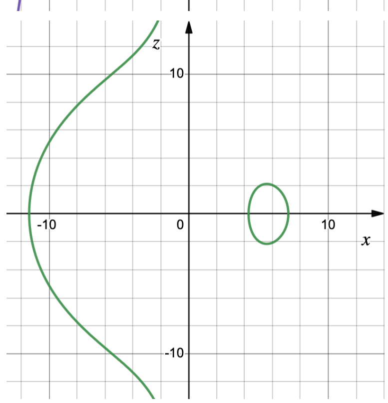
(iii)



(iv)

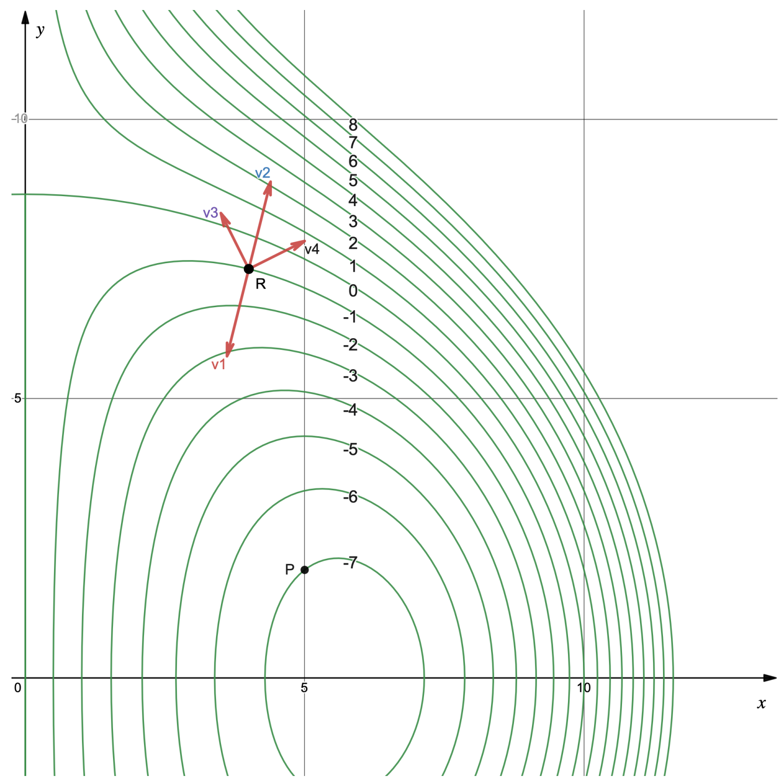


(v)



(f) A portion of the graph is reproduced below with four additional vectors $v1$, $v2$, $v3$, and $v4$ drawn on it. Which of the four vectors is $\nabla f(R)$?

- ☐ $v1$
- ☐ $v2$
- ☐ $v3$
- ☐ $v4$



Problem 2 (6 + 6 + 6 points)**.** Consider the vector $\vec{v} = \langle 2, -6, 9 \rangle$.

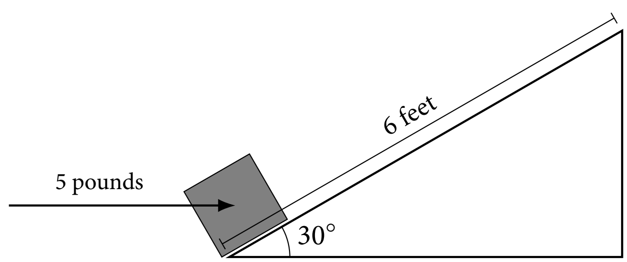
(a) Find a unit vector parallel to $\vec{v}$.

(b) Give an example of a vector $\vec{u} \neq \vec{v}$ that makes an obtuse angle with $\vec{v}$. *Briefly* explain your answer.

(c) Give an equation for a plane to which $\vec{v}$ is a normal vector.

Problem 3 (10 points). Parametrize the line through the point $(3, 4, 5)$ that is normal to the plane $2x - 7y + z = 8$.

Problem 4 (8 points). A woman exerts a horizontal force of 5 pounds on a box as she pushes it all the way up a ramp that is 6 feet long and inclined at an angle of 30 degrees above the horizontal. Find, in foot-pounds, the work done on the box. Show your work.



Problem 5 (22 points). Consider the function

$$f(x,y) = x^3 - 3xy + y^2.$$

It has two critical points. Find them, and use the Second Derivative Test to attempt to classify each one. Show your work.

critical point #1: (,) $\left\{ \begin{array}{l} \square \text{ local min} \\ \square \text{ local max} \\ \square \text{ saddle point} \\ \square \text{ inconclusive} \end{array} \right.$

critical point #2: (,) $\left\{ \begin{array}{l} \square \text{ local min} \\ \square \text{ local max} \\ \square \text{ saddle point} \\ \square \text{ inconclusive} \end{array} \right.$

Problem 6 (12 points)**.** You are given the following information about a function $W(s,t) = F(u(s,t), v(s,t))$.

$$
\begin{aligned}
u(1,0) &= 1 & u_s(1,0) &= 3 & u_t(1,0) &= 6\\
v(1,0) &= 3 & v_s(1,0) &= 5 & v_t(1,0) &= -5\\
F_u(1,3) &= 4 & F_v(1,3) &= 9 & &
\end{aligned}
$$

(a) Write down a Chain Rule for computing $W_s(1,0)$. (Do not compute it yet.)

(b) Now compute $W_s(1,0)$. Show your work.

Problem 7 (6 + 6 + 6 points). An unevenly heated metal plate has temperature $T(x, y)$ in degrees Celsius at a point (x, y). Suppose that $T(2, 1) = 145$, $T_x(2, 1) = 6$, and $T_y(2, 1) = -8$.

(a) Use the linearization to estimate the temperature at the point $(2.04, 0.95)$.

(b) Find (exactly) the instantaneous rate of change of T at $(2, 1)$ in the direction $\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.

(c) At $(2, 1)$, in which direction is the temperature increasing most rapidly? Give your answer as a unit vector.