These are the solutions to the white exam. The blue exam has the same problems, indicated in circles



NAME: answers

This exam should have 4 pages; please check that it does. Calculators (or any other electronic devices) are not allowed. **Show all work that you want considered for grading. An answer will only be counted if it is supported by all the work necessary to get that answer.** Give exact answers; for instance, don't give 3.14159 when the answer is $\pi$. Simplify as much as you can except if stated otherwise. No cheating.

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1. (12 points) Let $P = (1, 0, 1)$, $Q = (2, -1, 3)$ and $R = (0, 0, 3)$.

Blue #3

(a) Find parametric equations for the line between $P$ and $Q$. $\vec{PQ} = \langle 1, -1, 2 \rangle$

$x(t) = 1 + t$
$y(t) = 0 - t$
$z(t) = 1 + 2t$

(b) Find a normal vector to the plane containing $P$, $Q$ and $R$. (Hint: first find two vectors in the plane)

$\vec{PQ} = \langle 1, -1, 2 \rangle$ $\quad\quad$ $\vec{PR} = \langle -1, 0, 2 \rangle$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ -1 & 0 & 2 \end{vmatrix} = \vec{i}(-2) - \vec{j}\,4 + \vec{k}(-1)$$

(c) What is the area of the triangle with vertices $P$, $Q$ and $R$.

$\frac{1}{2} \| \vec{PQ} \times \vec{PR} \| = \frac{1}{2}\sqrt{4 + 16 + 1} = \frac{1}{2}\sqrt{21}$

2. (5 points) Find all vectors in 2 dimensions having $\|\vec{v}\| = 8$ where the $\vec{j}$-component is $4\vec{j}$.

$\vec{v} = a\vec{i} + 4\vec{j} \Rightarrow \|\vec{v}\| = \sqrt{a^2 + 16} = 8 \Rightarrow a = \sqrt{48}$ and $-\sqrt{48}$

Blue #2

3. (18 points) Match the surfaces with the verbal description of the level curves by placing the letter of the verbal description to the left of the number of the surface.

E (a) $z = 2x^2 + 3y^2$

A (b) $z = x^2 + y^2$

B (c) $z = \frac{1}{x+1}$

F (d) $z = 2x + 3y$

C (e) $z = \sqrt{x^2 + y^2}$

D (f) $z = xy$

Blue #1

- A. a collection of unequally spaced concentric circles
- B. a collection of unequally spaced parallel lines
- C. a collection of equally spaced concentric circles
- D. two straight lines and a collection of hyperbolas
- E. a collection of concentric ellipses
- F. a collection of equally spaced parallel lines

4. (12 points) Let $w = x^2 + 3y^2 + 2z^2$, $x = t^2 + 2$, $y = \frac{1}{t+1}$, $z = e^{2t}$.

(a) Write down the chain rule for $\frac{dw}{dt}$.

Blue #4

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

(b) Find $\frac{dw}{dt}$ when $t = 0$.

$$\frac{dw}{dt} = 2x(2t) + 6y\left(\frac{-1}{(t+1)^2}\right) + 4z\,2e^{2t}$$

$t=0 \Rightarrow x=2,\ y=1,\ z=1$

$$= 0 + 6\left(\frac{-1}{1}\right) + 4\cdot 2 = 2$$

Blue #7

5. (25 points) Let $g(x, y) = x^3y^2$.

(a) Write down the local linear approximation for $g$ near $(1, 3)$.

$g_x = 3x^2y^2 \qquad g_x(1,3) = 27$

$g_y = 2x^3y \qquad g_y(1,3) = 6$

$9 + 27(x-1) + 6(y-3)$

(b) Approximate $g(.8, 3.1)$ using the local linear approximation. You do not need to simplify your answer.

$9 + 27(-.2) + 6(.1)$

(c) The equation $z = g(x, y)$ describes a surface $S$ which contains the point $(1, 3, 9)$. Find an equation of the tangent plane to the surface $S$ at $(1, 3, 9)$.

$z = 9 + 27(x-1) + 6(y-3)$

(d) What is the value of the directional derivative $D_{\vec{u}}g(1, 3)$, where the unit vector $\vec{u}$ is in the direction of $-3\vec{\imath} + \vec{\jmath}$.

$\vec{u} = \frac{\langle -3, 1\rangle}{\sqrt{10}} \qquad \vec{\nabla} g(1,3) = \langle 27, 6\rangle$

$D_{\vec{u}}\, g(1,3) = \langle 27, 6\rangle \cdot \frac{\langle -3, 1\rangle}{\sqrt{10}} = \frac{-81+6}{\sqrt{10}} = \frac{-75}{\sqrt{10}}$

6. (10 points)

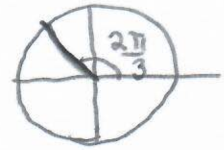
Blue #5

(a) For all values of the scalar $a$ is $< 3a, -2 >$ parallel to $< a^2, 1 >$.

$$\langle 3a, -2\rangle = c\langle a^2, 1\rangle \Rightarrow 3a = ca^2, \quad -2 = c$$
$$\Rightarrow 3a^2 = -2a^2 \Rightarrow a = 0, -\tfrac{3}{2}$$

(b) Suppose $\|\vec{v}\| = 3$, $\|\vec{w}\| = 4$ and the angle between $\vec{v}$ and $\vec{w}$ is $2\pi/3$. Find $\vec{v} \cdot \vec{w}$.

$$\vec{v} \cdot \vec{w} = 3 \cdot 4 \cos\tfrac{2\pi}{3} = 12\left(-\tfrac{1}{2}\right) = -6$$




7. (18 points) Let

$$f(x, y) = \frac{x^3}{3} + \frac{y^2}{2} - xy - 2y + 5.$$

Blue #6

(a) Find all critical points of $f(x, y)$.

$$f_x = x^2 - y = 0 \Rightarrow y = x^2$$
$$f_y = y - x - 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$$
$$\Rightarrow x = 2, -1$$
$$y = x^2 \qquad y = 4 \quad y = 1$$

$(2,4), (-1,1)$

(b) Classify each critical point as a local maximum, a local minimum, or a saddle point.

| | $(2,4)$ | $(-1,1)$ |
|---|---|---|
| $f_{xx} = 2x$ | 4 | $-2$ |
| $f_{yy} = 1$ | 1 | 1 |
| $f_{xy} = -1$ | $-1$ | $-1$ |
| $D = f_{xx} f_{yy} - f_{xy}^2$ | $3 > 0$ | $-3$ |
| | local min | saddle |