# MATH 208, EXAM 2

SECTION 250

Name: ______________________ NUID: ______________________

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**Instructions.**

- You should have 8 pages on which 7 problems are printed.
- You have 50 minutes: the exam will begin on the half hour and end promptly, 50 minutes later.
- Show all work unless otherwise specified. What you write on the page must convince me that you understand the problem and its solution.
- Read each problem carefully.
- You do not need to simplify your answers, unless the instructions for a problem indicate otherwise.
- You are not allowed a calculator, notes, textbooks, or access to any electronic devices.
- Don't panic. Good luck!

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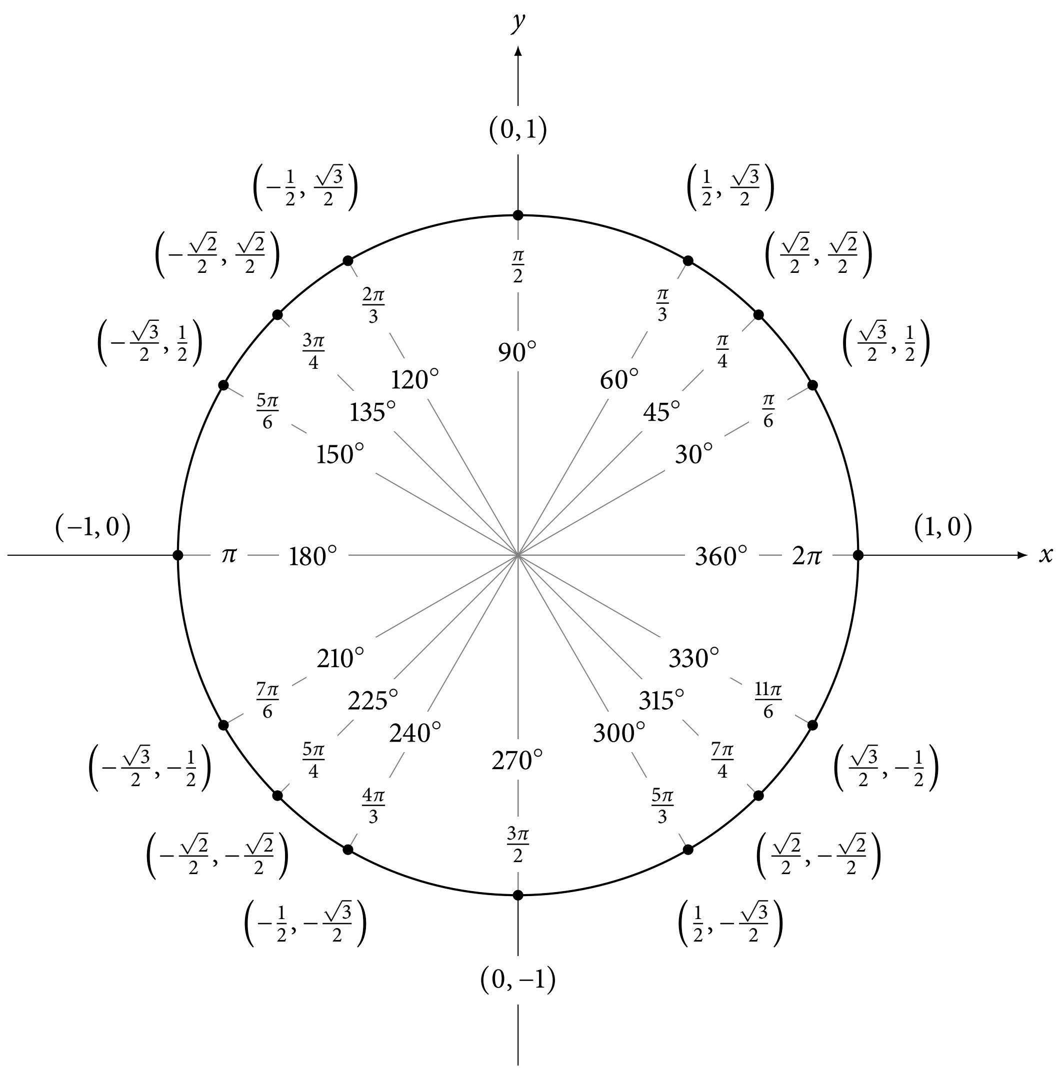
*Date*: Fall 2022.

Here are some things you might find useful.

$$
\begin{aligned}
x &= \rho \sin\phi \cos\theta \\
y &= \rho \sin\phi \sin\theta \\
z &= \rho \cos\phi
\end{aligned}
$$

$$dV = \rho^2 \sin\phi \, d\rho d\phi d\theta$$



**Problem 1** (8 points)**.** Values of $f(x, y)$ are shown in a table below. Let $R$ be the rectangle $[2, 2.6] \times [5, 5.2]$, i.e., all points $(x, y)$ satisfying $2 \le x \le 2.6$ and $5 \le y \le 5.2$. Use the table of values to produce a reasonable *underestimate* for the integral $\iint_R f(x, y)\, dA$. (**Do not simplify your answer.**)

| | $x = 2$ | $x = 2.2$ | $x = 2.4$ | $x = 2.6$ |
|---|---|---|---|---|
| $y = 5$ | 4 | 3 | −1 | −5 |
| $y = 5.1$ | 0 | 2 | 1 | 3 |
| $y = 5.2$ | −1 | −2 | −3 | −4 |

**Problem 2** (8 points)**.** A biologist is studying a population of microorganisms in a circular petri dish of radius 5cm. Her model predicts that the population density of microorganisms should be $\frac{C}{e^r}$ individuals/cm$^2$ at distance $r$ from the center of the dish, for some constant $C$. Set up (**but do not solve**) an integral giving the total population of microorganisms in the dish.

**Problem 3** (20 points). Use the method of Lagrange multipliers to find the maximum value taken by the function $f(x, y, z) = 3yz - x^2$ on the plane $-2x + 6y + 6z = 11$.

Maximum: $f($ ☐ , ☐ , ☐ $) =$ ☐ .

**Problem 4** (20 points). A thick tube is bounded below by the $xy$-plane, above by the plane $z = 2x + 5$, and between the two cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$.

(a) Set up an integral in cylindrical coordinates to compute the volume of this region.

(b) Evaluate your integral.

**Problem 5** (6 + 6 + 6 points)**.** For each of the following regions $W$, set up (**but do not evaluate**) an iterated integral in spherical coordinates giving $\iiint_W z\,dV$. (The integrand is the function $f(x, y, z) = z$.)

(a) $W$ is the region between the two spheres $x^2 + y^2 + z^2 = 3$ and $x^2 + y^2 + z^2 = 16$.

(b) $W$ is the set of points $(x, y, z)$ lying between the two spheres $x^2 + y^2 + z^2 = 3$ and $x^2 + y^2 + z^2 = 16$ and satisfying $y \geq 0$, $z \leq 0$.

(c) $W$ is the set of points $(x, y, z)$ lying between the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 16$.

**Problem 6** (6 + 6 points). Let $T$ be the tetrahedral region bounded by the $xy$-plane, the $xz$-plane, the $yz$-plane, and the plane $x + 3y + 5z = 15$. Express the integral $\iiint_T f\,dV$ as an iterated integral using the two requested orders of integration.

(a)

$$\int_{x=} \int_{y=} \int_{z=} f(x,y,z)\,dz\,dy\,dx$$

(b)

$$\int_{z=} \int_{x=} \int_{y=} f(x,y,z)\,dy\,dx\,dz$$

**Problem 7** (7 + 7 points)**.** The two parts are independent.

(a) Let $L$ be the line that is the intersection of the planes $z = 3$ and $x = y$. Notice that the two points $(5, 5, 3)$ and $(0, 0, 3)$ each lie on $L$. Give a parametrization $\vec{r}(t)$ of $L$ satisfying $\vec{r}(0) = \langle 5, 5, 3\rangle$ and $\vec{r}(2) = \langle 0, 0, 3\rangle$.

(For partial credit, give any correct parametrization of $L$.)

(b) Give a vector-valued function that parametrizes the circle of radius 5 centered at $(-3, 4)$ in a **clockwise** direction.

(For partial credit, give any correct parametrization of this circle.)