NAME: Solutions

Circle which recitation section you are in:

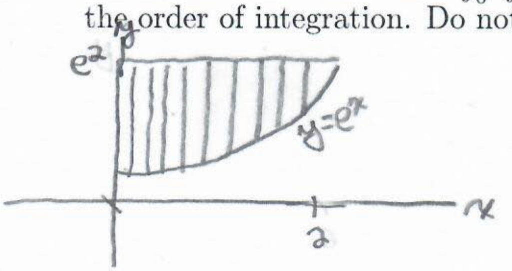
Audrey 9:30 Audrey 11:00 Nick 12:30 Audrey 2:00

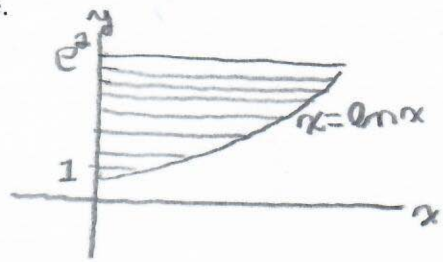
This exam should have 4 pages; please check that it does. Nothing besides pen and/or pencil and/or eraser will be allowed during the exam. **Show all work that you want considered for grading. An answer will only be counted if it is supported by all the work necessary to get that answer.** Give exact answers; for instance, don't give 3.14159 when the answer is π. Simplify as much as you can except if stated otherwise. No cheating.

The following might (or might not) be useful in this exam:

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho\cos\phi, \quad dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$

1. (15 points) For the integral $\int_0^2 \int_{e^x}^{e^2} dy\, dx$, draw the region of integration. Then reverse the order of integration. Do not evaluate.





#4 in Exam 2B

$$\int_1^{e^2} \int_0^{\ln x} dx\, dy$$

2. (9 points) Let W be the *bottom* half of the ball $x^2 + y^2 + z^2 \leq 10$. Without evaluating, state whether the integrals are positive, negative or zero. For each answer give a brief reason.

#2 in Exam 2B

(a) $\iiint_W z\, dV$. $z \leq 0$ on $W \Rightarrow$ negative

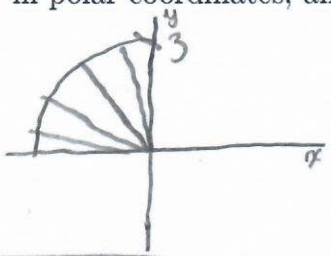
(b) $\iiint_W (x+y)\, dV$. cancellation of x's and y's $\Rightarrow 0$

(c) $\iiint_W (x+y)^2\, dV$. $(x+y)^2 > 0 \Rightarrow$ positive

3. (14 points) Let R be that part of the region inside $x^2+y^2=9$ which has $x<0$ and $y>0$. Write the integral

$$\int\int_R (x^2+y^2)^{3/2}dA$$

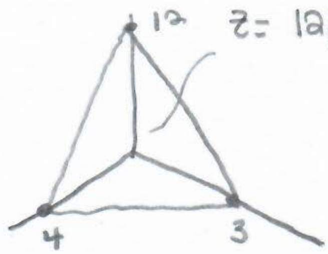
in polar coordinates, and evaluate the integral.



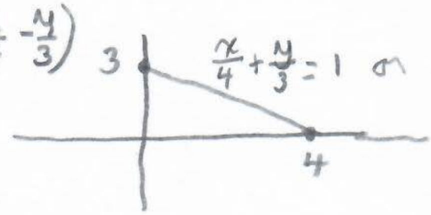
$$\int_{\pi/2}^{\pi}\int_0^3 r^3 r\,dr\,d\theta = \int_{\frac{\pi}{2}}^{\pi}\left(\frac{r^5}{5}\Big|_0^3\right)d\theta = \frac{\pi}{2}\frac{3^5}{5} = \frac{243\pi}{10}$$

#1 in Exam 2B

4. (14 points) Let W be the region cut off by the plane $\frac{x}{4}+\frac{y}{3}+\frac{z}{12}=1$ which is in the first octant (reminder: the first octant has $x>0$, $y>0$ and $z>0$.). Write a triple integral for the volume of W in Cartesian coordinates. Do not evaluate.



$z = 12\left(1-\frac{x}{4}-\frac{y}{3}\right)$



$\frac{x}{4}+\frac{y}{3}=1$ or $y=3\left(1-\frac{x}{4}\right)$

$$\int_0^4\int_0^{3(1-\frac{x}{4})}\int_0^{12(1-\frac{x}{4}-\frac{y}{3})} dz\,dy\,dx$$

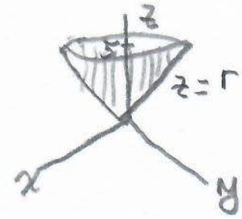
#3 in Exam 2B

You can set it up in any order of integration.

5. (20 points) Let W be the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and bounded above by the plane $z = 5$.

(a) Set up a cylindrical integral for the volume of W. Do not evaluate.

$$\int_0^{2\pi} \int_0^5 \int_r^5 dz\, r\,dr\,d\theta$$



#7 in Exam 2B

(b) Set up a spherical integral for the volume of W. Do not evaluate. Hint: First find the spherical equation for the plane $z = 5$.

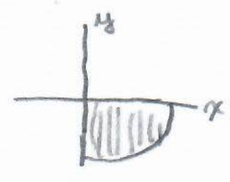
$$\rho \cos\phi = 5 \Rightarrow \rho = \frac{5}{\cos\phi}$$

$z=r$ so $\phi = \frac{\pi}{4}$

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\frac{5}{\cos\phi}} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

(c) Suppose S is the solid that is the part of W which has $x \geq 0$ and $y \leq 0$. Set up a cylindrical integral for the volume of S. Do not evaluate.

$$\int_{\frac{3\pi}{2}}^{2\pi} \int_0^5 \int_r^5 dz\, r\,dr\,d\theta$$



6. (10 points) (a) Find parametric equations in three dimensions for the circle of radius 4 in the plane $z = 5$ centered at $(0, 1, 5)$. Set it up so that for $0 \le t < 2\pi$ the circle is traversed exactly once.

#6 on Exam 2B

$x = 0 + 4\cos t$
$y = 1 + 4\sin t$
$z = 5$

$0 \le t \le 2\pi$

(b) For the circle in part (a), modify the parametric equations so that for $0 \le t < 1$ the circle is traversed exactly once.

$x = 4\cos(2\pi t)$
$y = 1 + 4\sin(2\pi t)$
$z = 5$

$0 \le t \le 1$

7. (18 points) We will use Lagrange multipliers to find the mimimum and maximum of the function $x + y - z$ on the ellipsoid $x^2 + 2y^2 + 4z^2 = 28$.

(a) Identify all (x, y, z) that satisfy the Lagrange multiplier equations.

#5 on Exam 2B

$1 = 2x\lambda$
$1 = 4y\lambda$
$-1 = 8z\lambda$

$\Rightarrow 2x = 4y = 8z$ or $x = 2y = 4z$

$x^2 + 2y^2 + 4z^2 = 28 \Rightarrow (4z)^2 + 2(2z)^2 + 4z^2 = 28$

$\Rightarrow z^2 = 1 \Rightarrow z = \pm 1$

$\Rightarrow (-4, -2, 1)$ and $(4, 2, -1)$

(b) Find the max and min for the problem. Give the value of the function, and the point (or points) at which the max and min are taken on.

$\underbrace{f(-4,-2,1) = -7}_{\text{min}}$ $\qquad$ $\underbrace{f(4,2,-1) = 7}_{\text{max}}$