MATH 208, EXAM 3

SECTION 150

Name: ______________________________ NUID: ______________________________

Instructions.

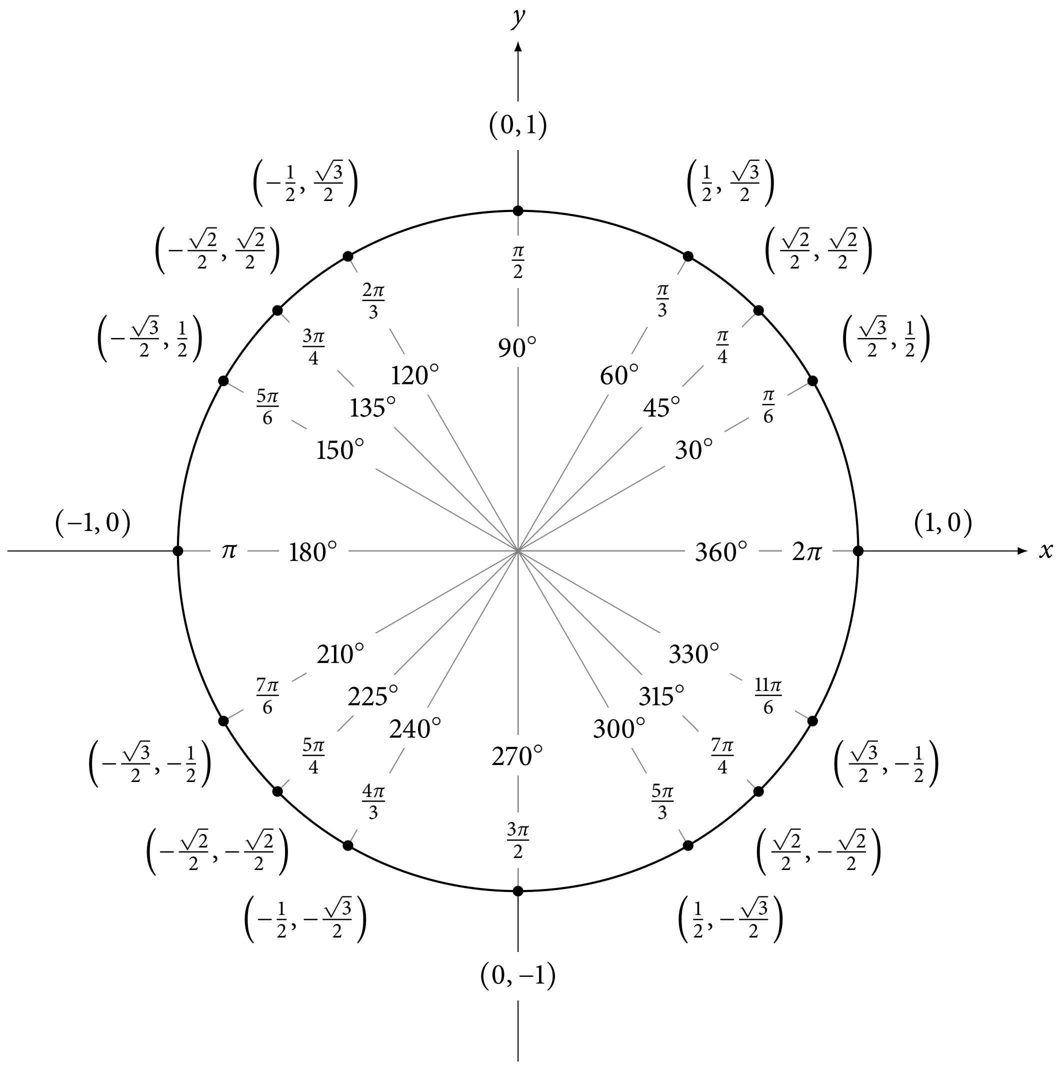
- You should have 8 pages on which 6 problems are printed.
- You have 50 minutes: the exam will begin on the half-hour and end promptly, 50 minutes later.
- Show all work unless otherwise specified. What you write on the page must convince me that you understand the problem and its solution.
- Read each problem carefully.
- You do not need to simplify your answers, unless the instructions for a problem indicate otherwise.
- You are not allowed a calculator, notes, textbooks, or access to any electronic devices.
- Don't panic. Good luck!

Date: Fall 2022.

Here are some things you might find useful.

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dV = \rho^2 \sin\phi \, d\rho d\phi d\theta$$



Problem 1 (2 + 8 points). Consider the spiral curve parametrized by

$$\vec{r}(t) = \langle 3t, 4\sin t, 4\cos t\rangle \quad \text{for} \quad -1 \le t \le 3.$$

(a) Which of the following correctly describes the line tangent to this curve at the point $(0, 0, 4)$? (There is exactly one correct answer.)

- ☐ $3x + 4y + z = 0$
- ☐ $x = 3t,\ y = 4\sin t,\ z = 4\cos t$
- ☐ $x = 3t,\ y = 4t,\ z = 4$
- ☐ $\vec{L}(t) = \langle 0, 0, 0\rangle + t\langle 0, 0, 4\rangle$

(b) Find the arclength of the spiral curve.

Problem 2 (8 + 12 points)**.** Let S be the portion of the cylinder $x^2 + y^2 = 4$ lying above the xy-plane and below the plane $z = y$.

(a) Give a parametrization of the surface S. Be sure to give bounds on your parameters.

(b) Express the surface area of S as a double integral. **Do not evaluate your integral.**

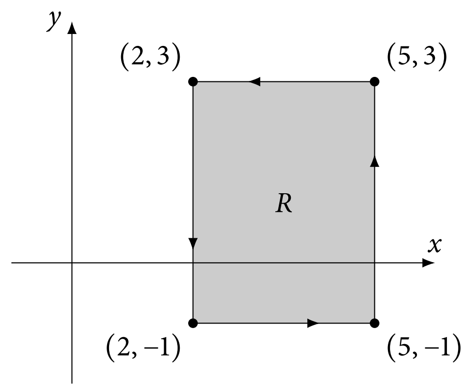
Problem 3 (15 + 5 points). Consider the following vector field in three dimensions.

$$\vec{F} = \left\langle e^{-2y}, 3z - 2xe^{-2y}, 3y \right\rangle$$

(a) Clearly state what it means for g to be a potential function for $\vec{F}$, and then find such a potential function g. (Show your work.)

(b) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ over any path C from $(1, 0, \pi)$ to $(0, 1, 2)$. Briefly explain why your answer depends only on the endpoints of C and not on its behavior between the endpoints.

Problem 4 (10 points)**.** Consider the rectangle R in the plane with vertices $(2,-1)$, $(5,-1)$, $(5,3)$, and $(2,3)$ and the vector field $\vec{F} = \left\langle 3y - e^{x^2}, e^{x^2+y^2} \right\rangle$. Use Green's Theorem to express the circulation of $\vec{F}$ around the boundary of R (oriented counter-clockwise) as a double integral. **Do not evaluate your integral.**



Problem 5 (20 points). Compute the line integral $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = \langle -2e^y, -xz, xy\rangle$ and C is the straight-line path from $(1,0,0)$ to $(0,2,3)$.

Problem 6 (20 points). Compute the flux of the vector field $\vec{F} = \langle -x, 2y, 3z \rangle$ over the portion of the plane $3x + 3z = 10$ that lies above the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$, oriented with upward-pointing normal vector.