NAME: Key

The instructions on the exam will be the following:

Circle which recitation section you are in:

Audrey 9:30 Audrey 11:00 Nick 12:30 Audrey 2:00

This exam should have 6 pages; please check that it does. Nothing besides pen and/or pencil and/or eraser will be allowed during the exam. **Show all work that you want considered for grading. An answer will only be counted if it is supported by all the work necessary to get that answer.** Give exact answers; for instance, don't give 3.14159 when the answer is π. Simplify as much as you can except if stated otherwise. In multi-part problems, the points are not necessarily the same number of points. No cheating.

The following might (or might not) be useful in this exam:

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho\cos\phi, \quad dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$

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1. (12 points) Find the work done by the field $\vec{F} = \langle 1 + y, x^2 \rangle$ on a particle that goes along the straight line segment from $(0, 0)$ to $(1, -2)$.

$x = t \quad dx = dt \quad t: 0 \to 1$

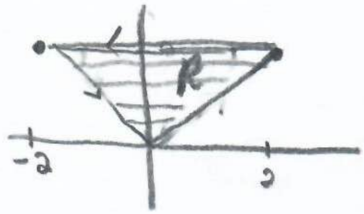
$y = -2t \quad dy = -2dt$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 (1-2t)dt + t^2(-2dt) = \int_0^1 (1-2t-2t^2)dt$$

$$= \left(t - t^2 - \frac{2}{3}t^3\right)\Big|_0^1 = \left(1 - 1 - \frac{2}{3}\right) - 0 = -\frac{2}{3}$$

2. (16 points) Let C be the triangle with vertices $(0,0)$, $(-2,2)$ and $(2,2)$, traversed counterclockwise. Use Green's Theorem to evaluate

$$\int_C -y^2\,dx + (xy)dy.$$



$$= \iint_R [y-(-2y)]\,dA = \int_0^2 \int_{-y}^{y} 3y\,dx\,dy$$

$$= \int_0^2 3y(2y)dy = 6\frac{y^3}{3}\Big|_0^2 = 16$$

3. (12 points) Find a function f so that

$$\vec{\nabla} f = (ye^{xy} + x^2)\vec{i} + (1 + xe^{xy})\vec{j}.$$

Either show how you got your answer, or verify that your answer is correct.

$f_x = ye^{xy} + x^2 \Rightarrow f = \int (ye^{xy} + x^2)\,dx = \frac{ye^{xy}}{y} + \frac{x^3}{3} + \frac{y}{\text{no } x}$

$f_y = 1 + xe^{xy} \Rightarrow f = \int (1 + xe^{xy})\,dy = y + \frac{xe^{xy}}{x} + \frac{x^3/3}{\text{no } y}$

$f = e^{xy} + \frac{x^3}{3} + y$

4. (12 points) Let $\vec{F} = yz\vec{i} + xz\vec{j} + zy\vec{k}$, so $\vec{F} = \vec{\nabla} f$, where $f = xyz$. Let C be the parameterized curve

$$x = 2t + 1, \quad y = \cos(\pi t), \quad z = 2t+3, \quad 0 \le t \le 1/2.$$

Find $\int_C \vec{F} \cdot d\vec{r}$. Hint: There's an easy way and a hard way.

$= f(\text{end}) - f(\text{start}) = 0 - 3 = -3$

start at $t=0$ is $(1,1,3)$

end at $t=\frac{1}{2}$ is $(2,0,4)$

5. (16 points) For the following surfaces, give the parametrization using the parameters t and s, making sure to show the range of the parameters.

(a) Find parametric equations for the top half of the sphere centered at the origin and with radius 6. By "top half" we mean the part with $z > 0$.

$x = 6\sin t\cos s$ $\quad 0 \le t \le \frac{\pi}{2}$

$y = 6\sin t\sin s$ $\quad 0 \le s \le 2\pi$

$z = 6\cos t$

$\boxed{\text{OR}}$

$x = t\cos s$ $\quad 0 \le s \le 2\pi$

$y = t\sin s$ $\quad 0 \le t \le 6$

$z = \sqrt{36 - t^2\cos^2 s - t^2\sin^2 s}$

$= \sqrt{36 - t^2}$

(b) Find parametric equations for the part of the plane $x + y + z = 9$ which is in the the cylinder $x^2 + y^2 = 25$.

$x = s\cos t$ $\quad 0 \le s \le 5$

$y = s\sin t$ $\quad 0 \le t \le 2\pi$

$z = 9 - s\cos t - s\sin t$

6. (14 points) For each of the descriptions, give a possible vector field in two dimensions which matches the description:

   (a) Every vector in the vector field has length 1, and points towards the origin.

   $$\vec{F} = -\frac{\langle x, y\rangle}{\sqrt{x^2+y^2}}$$

   (b) For $x < 0$, the vectors point straight up, for $x > 0$ the vectors point straight down.

   $\vec{F} = -x\vec{j}$ there are many answers

   (c) The vectors rotate counterclockwise.

   $\vec{F} = \langle -y, x\rangle$ there are many answers

7. (18 points) Let $\vec{F} = \langle 3x, 3y, z\rangle$. Let S be the surface with parameterization

$$x = \cos 2t, \quad y = \sin 2t, \quad z = s, \quad 0 \le t \le \pi/2, \quad 0 \le s \le 2.$$

(a) Find $\vec{r}_s \times \vec{r}_t$.

$\vec{r}(s,t) = \langle \cos 2t, \sin 2t, s\rangle$

$\vec{r}_s = \langle 0 \quad 0 \quad 1\rangle$

$\vec{r}_t = \langle -2\sin 2t \quad 2\cos t \quad 0\rangle$

$\vec{r}_s \times \vec{r}_t = \langle -2\cos 2t, -2\sin 2t, 0\rangle$

(b) Find the surface area of S.

$\|\vec{r}_s \times \vec{r}_t\| = \sqrt{4(\cos^2 t + \sin^2 t)} = 2$

Area $= \int_0^2 \int_0^{\pi/2} 2 \, ds \, dt = 2(2)\frac{\pi}{2} = 2\pi$

(c) Find the flux of $\vec{F}$ over S if S is oriented with normals that have positive $\vec{j}$ component when x and y are positive.

Flux $= \pm \int_0^2 \int_0^{\pi/2} \langle 3x, 3y, z\rangle \cdot \langle -2\cos 2t, -2\sin 2t, 0\rangle \, dt \, ds$

need $-$ to make normals have positive $\vec{j}$ component

$= \int_0^2 \int_0^{\pi/2} \langle 3\cos 2t, 3\sin 2t, s\rangle \cdot \langle 2\cos 2t, 2\sin 2t, 0\rangle \, dt \, ds$

$= \int_0^2 \int_0^{\pi/2} 6 \, dt \, ds = 6(2)\left(\frac{\pi}{2}\right) = 6\pi$