Math 208 Final Exam Initials:

Name (write clearly) ______________________

NU ID number ______________________

Circle the name and time of your lecture:

Norwood 8:30 Norwood 9:30 Schafhauser 11:30 Rebarber 12:30 Burns 6:30 Yang 208H.

Question	Out of	Score
1	10	
2	10	
3	12	
4	12	
5	12	
6	12	
7	16	
8	12	
9	18	
10	16	
11	13	
12	12	
13	14	
14	17	
15	14	
TOTAL	200	

Instructions

- There are **15** questions on **17** pages (including this cover sheet and the formula sheet on the second page).
- No books, notes or calculator are allowed.
- Turn off all communication devices.
- **Show all your work** and explain your answers. Unsupported answers will receive **little credit**.
- If specified, use the method required by each problem. Alternate methods will not receive full credit.
- In multi-part problems, the parts might not be worth the same number of points.
- You have **2 hours** to complete the exam.

Good luck !

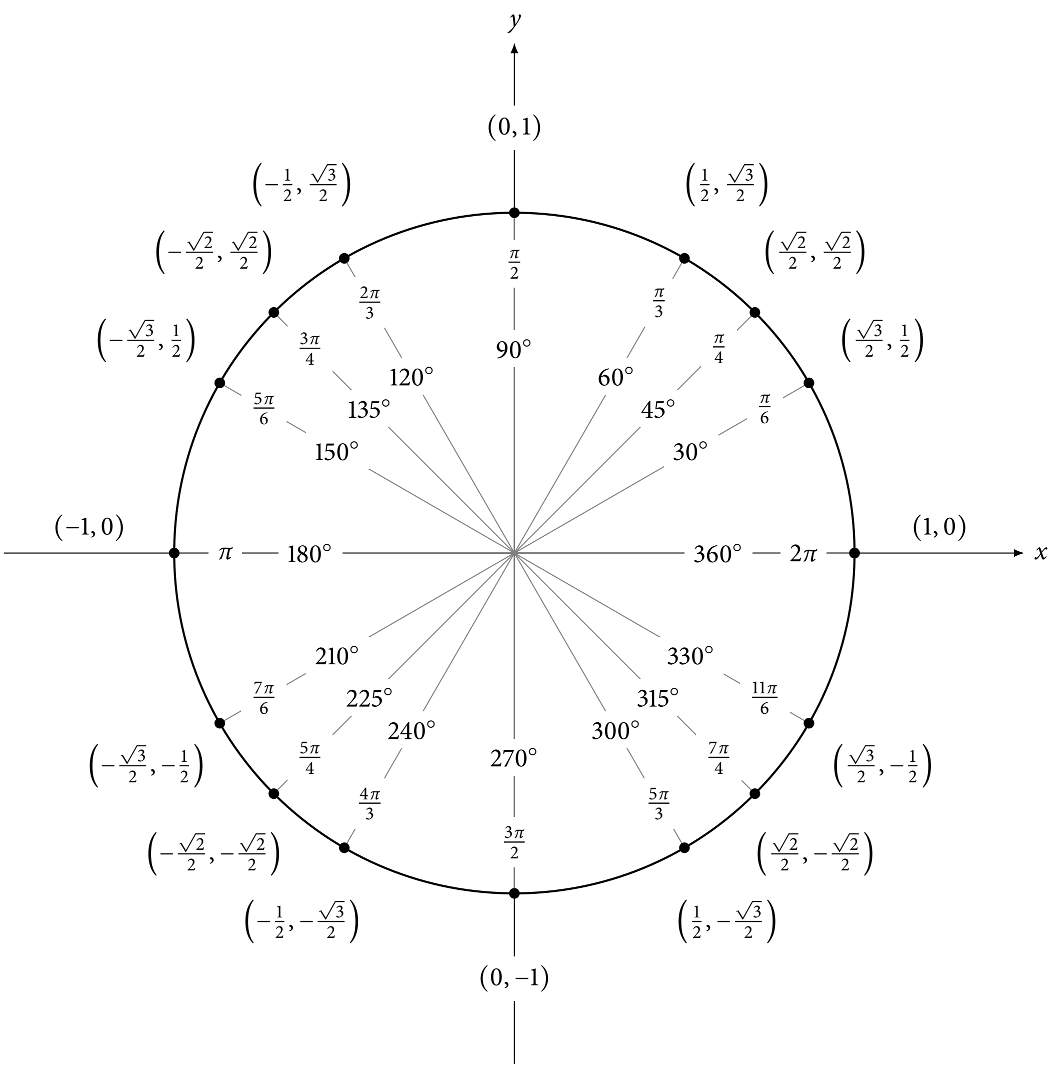
The formulas on the next page might or might not be useful:

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi, \quad dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta.$$

$$\int_C \vec{F} \cdot d\vec{r} = \pm \int \int \text{curl}(\vec{F}) \cdot (\vec{r}_s \times \vec{r}_t) \, ds \, dt$$

The flux of $\vec{F}$ over the boundary of W is equal to the integral of the divergence of $\vec{F}$ over W.

$$D = f_{xx} f_{yy} - f_{xy}^2.$$



1. (10 points)

   (a) Find a normal vector to the plane containing the points $(0,1,3)$, $(-2,0,-1)$ and $(1,1,0)$.

   (b) Find an equation for the plane in part (a).

2. (10 points) Match each vector field formula with the corresponding graph.

(i) $\langle 0, x\rangle$

- ☐ Graph A
- ☐ Graph B
- ☐ Graph C
- ☐ Graph D
- ☐ Graph E

(ii) $\langle 0, y\rangle$

- ☐ Graph A
- ☐ Graph B
- ☐ Graph C
- ☐ Graph D
- ☐ Graph E

(iii) $\langle x, y\rangle$

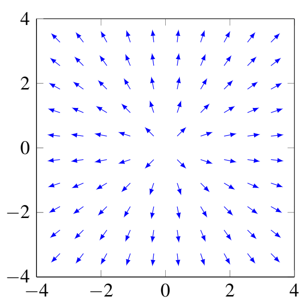
- ☐ Graph A
- ☐ Graph B
- ☐ Graph C
- ☐ Graph D
- ☐ Graph E

(iv) $\frac{1}{\sqrt{x^2+y^2}}\langle x, y\rangle$

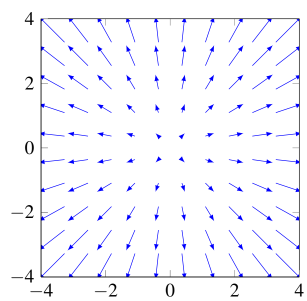
- ☐ Graph A
- ☐ Graph B
- ☐ Graph C
- ☐ Graph D
- ☐ Graph E

(v) $\langle |\sin(y)|, 0\rangle$

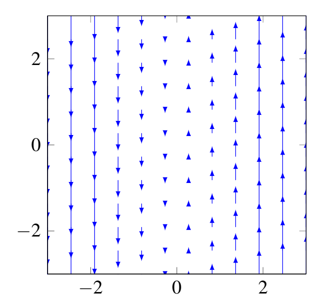
- ☐ Graph A
- ☐ Graph B
- ☐ Graph C
- ☐ Graph D
- ☐ Graph E



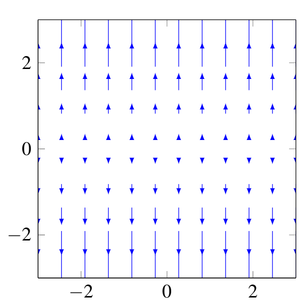
Graph A



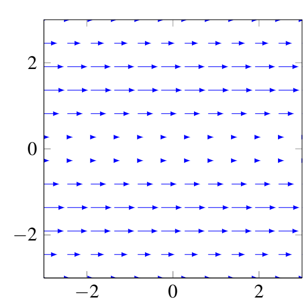
Graph B



Graph C



Graph D



Graph E

3. (12 points)

(a) Find a number α such that $2\vec{i}+\vec{j}+3\vec{k}$ is perpendicular to $\alpha\vec{i}+2\vec{j}-6\vec{k}$.

(b) Find the **cosine** of the angle between the two vectors $\vec{u}=-2\vec{i}+3\vec{j}+6\vec{k}$ and $\vec{v}=\vec{i}+2\vec{j}-\vec{k}$.

4. (12 points)

(a) Find the directional derivative of $f(x,y,z) = ze^y + x^2$ at $P = (5,0,-1)$ in the direction from the point P to the point $Q = (4,1,2)$.

(b) What is the direction of maximum rate of change of f at P? Give the direction as a unit vector.

5. (12 points)

(a) Draw the region of integration for

$$\int_1^{e^2} \int_0^{\ln y} dx\, dy.$$

(b) Switch the order of integration for this integral. Do not evaluate.

6. (12 points)

(a) Find the local linearization of $f(x,y) = (xy^2+7)^{3/2}$ at the point $(2,1)$.

(b) For $f(x,y)$ in part (a), use the local linearization to approximate $f(2.05,.9)$. Leave your answer as a number, but there is no need to simplify it.

7. (16 points) (a) Find both **critical points** of $f(x,y) = 3xy - x^3 - y^3 + 3$.

(b) Use the **Second Derivative Test** to classify each of the critical point(s) as a local maximum, local minimum, or saddle point.

8. (12 points) Use **Green's Theorem** to evaluate the line integral

$$\int_C (2xy-3)dx + (x^2+x+y)dy,$$

where C is the close curve consisting of the line segment from $(0,0)$ to $(2,0)$, followed by the line segment from $(2,0)$ to $(0,3)$, followed by the line segment from $(0,3)$ back to $(0,0)$.

9. (18 points) Let W be the part of the solid ball $x^2 + y^2 + z^2 \leq 16$ which is in the octant with

$$\{x \geq 0,\ y \leq 0,\ z \leq 0\}.$$

For the following problems, you only get credit for providing the integrals.

(a) Find an integral for the volume of W in spherical coordinates. Do not evaluate.

(b) Find an integral for the volume of W in cartesian coordinates. Do not evaluate.

10. (16 points) Let S be that part of the surface $z = xy + 5$ which is above the square

$$\{0 \leq x \leq 1, \;\; 1 \leq y \leq 2\}$$

in the (x, y) plane. Assume S is oriented with normals that have a positive $\vec{k}$ component. Find the flux of the vector field $\vec{F}(x, y, z) = \langle x, 0, xy + 1 \rangle$ through S.

11. (13 points) Use **Lagrange multipliers** to find the maximum and minimum values (and the points (x,y) where they are taken on) of $f(x,y) = x + 3y$ subject to the constraint $x^2 + y^2 = 4000$.

12. (12 points) Let $\vec{F}(x,y,z) = -3y\vec{i} + x\vec{j}$ and let C be that part of the curve $y = x^2$ from $(-1,1)$ to $(1,1)$. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$.

13. (14 points) Let R be the solid region bounded below by $z = x^2 + y^2$ and above by $z = 32 - (x^2 + y^2)$. Let S be the boundary surface of R. Use the **Divergence Theorem** to find an iterated integral, **in cylindrical coordinates**, for the outward flux of the vector field

$$\vec{F} = (x^3 + y^3)\vec{i} + (y^3 + xz)\vec{j} + (y - x^2 y)\vec{k}$$

through the surface S. Your answer should be in terms of z, r and θ, and you should not evaluate the iterated integral.

14. (17 points) Let C be the curve $x^2+y^2=25$ in the plane $z=1$, oriented counterclockwise when viewed from above. Let $\vec{F}=\langle z,x,y\rangle$. Let S be the disk enclosed by C, that is, the surface defined by $x^2+y^2\leq 25$ in the plane $z=1$.

(a) Find parametric equations for S, with parameters s and t.

(b) Use **Stokes' Theorem** to find a flux integral which is equivalent to

$$\int_C \vec{F}\cdot d\vec{r}.$$

Write the integral completely in terms of s and t. Do not evaluate the integral.

15. (14 points) (a) Show that this vector field is conservative:

$$\vec{F}(x,y) = \left(\frac{2x}{y} + 2x\right)\vec{i} + \frac{-x^2}{y^2}\vec{j}.$$

(b) Find a potential function for the vector field in part (a).

(c) Find the work done by the force field in part (a) in moving an object from $(2,5)$ along a curve C to $(2,1)$.