Name ______________________________

Circle the name of your instructor:

GIUSTI	JIN	OLSEN	PETERSON	RUIZ	SECELEANU	SHULTIS
10:30 am	12:30 pm	6:30 pm	8:30 am	12:30 pm	9:30 am	11:30 am

Instructions

- There are **16** questions on **9** pages (including this cover sheet).
- No books or other notes are allowed.
- You may use a calculator.
- Turn off all communication devices.
- **Show all your work** and explain your answers. Unsupported answers will receive **little credit**.
- If specified, use the method required by each problem. Alternate methods will not receive full credit.
- You have **2 hours** to complete the exam.

Good luck !

Question	Out of	Score
1	12	
2	12	
3	13	
4	12	
5	12	
6	12	
7	12	
8	13	
9	13	
10	13	
11	13	
12	11	
13	13	
14	13	
15	13	
16	13	
TOTAL	200	

1. (12 points) Consider the vectors $\vec{v} = -\vec{i}$ and $\vec{w} = \vec{j} + \vec{k}$.

   (a) Compute the dot product $\vec{v} \cdot \vec{w}$.

   (b) Are the vectors $\vec{v}$ and $\vec{w}$ perpendicular to each other? Explain your answer.

   (c) Compute the magnitude of the second vector: $||\vec{w}||$.

   (d) Find the **unit** vector in the direction of $\vec{w}$.

2. (12 points) Let $P = (1,1,1)$, $Q = (3,5,-2)$, and $R = (-4,1,2)$.

   (a) Find the **displacement vectors** $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (b) Find the **cross product** $\overrightarrow{PQ} \times \overrightarrow{PR}$.

   (c) Find the equation of the plane passing through the points P, Q, and R.

3. (13 points) Find the **directional derivative** of $f(x,y) = \sin(xy) + 3xy^2$ in the direction of the vector $\vec{v} = \vec{i} + \vec{j}$ at the point $(2,3)$.

4. (12 points) Let $z = f(x,y) = x^2 - y$. Draw a contour diagram for f, for the fixed values $z = 0, 1, 2, 3$. Draw all the contours on the same diagram in the xy-plane. **Remember to label your axes!**

5. (12 points) Let $z = 2xy^2$, $x = ve^t$, and $y = t\sin(v)$. Use the **chain rule** to compute $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial v}$.

6. (12 points)

(a) Find the **critical points** of $f(x,y) = 5y + xy + x^2 - y^2 + 1$.

(b) Use the **Second Derivative Test** to classify each of the critical point(s) as a local maximum, local minimum, or saddle point.

7. (12 points) Set up a double integral that gives the mass of the plate bounded by the curves $x = y^2$ and $y = 2 - x$ given that the mass density is given by $\delta(x,y) = x^2 + y^2$. **Do not evaluate the integral.**

8. (13 points) Set up a triple integral to find the volume of the region above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 4$. **Do not evaluate the integral.**

9. (13 points) Sketch the region of the integration $\int_1^5 \int_x^{2x} \sin x\,dy\,dx$ in the xy-plane. Clearly shade this region. Rewrite the integral reversing the order the integration. **Do not evaluate the integral.**

10. (13 points) Use **Lagrange multipliers** to find the maximum and minimum values of $f(x,y) = x+3y+2$ subject to the given constraint $g(x,y) = x^2+y^2 = 10$, if such values exist.

11. (13 points) Find the **work** done by the force field $\vec{F}(x,y,z) = 3y\vec{i} + 4x\vec{j} - \vec{k}$ in moving an object along the line segment from $(2,1,1)$ to $(3,2,3)$.

12. (11 points) Find a **parametrization** of the plane containing the points $(1,1,1)$, $(-1,2,1)$ and $(0,0,3)$.

13. (13 points) Use the **Divergence Theorem** to find the outward flux of the vector field $\vec{F} = (x^3 + z)\vec{i} + y^3\vec{j} + (z^3 - y)\vec{k}$ through the surface S of the sphere $x^2 + y^2 + z^2 = 4$.

14. (13 points) Use **Stoke's Theorem** to find a **double integral** that gives the circulation of the vector field $\vec{F}(x,y,z) = z^2\vec{i} + 3xy\vec{j} + 5yz\vec{k}$ around the triangle from $P = (2,0,0)$, to $Q = (0,3,0)$, to $R = (0,0,6)$ and back to P (this triangle lies in the plane $3x+2y+z=6$). **Do not evaluate the integral.**

15. (13 points)

(a) Find a **potential** function for the vector field $\vec{F}(x,y) = 2x\vec{i} + 4y^3\vec{j}$.

(b) Let C_1 be the quarter of the circle $x^2 + y^2 = 4$ in the first quadrant, oriented in the counter-clockwise direction. Use your answer to evaluate the line integral $\int_{C_1} \vec{F} \cdot d\vec{r}$.

(c) Explain why $\int_{C_2} \vec{F} \cdot d\vec{r} = 0$ if C_2 is the full circle $x^2 + y^2 = 4$, oriented counterclockwise.

16. (13 points) Use **Green's Theorem** to evaluate the line integral

$$\int_C (\tan x + 2x^2y)dx + (y^2 + 2 - 2xy^2)dy,$$

where C is the circle $x^2 + y^2 = 9$ oriented in the clockwise sense. (In different notation, this problem is asking you to compute $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = (\tan x + 2x^2y)\vec{i} + (y^2 + 2 - 2xy^2)\vec{j}$.)