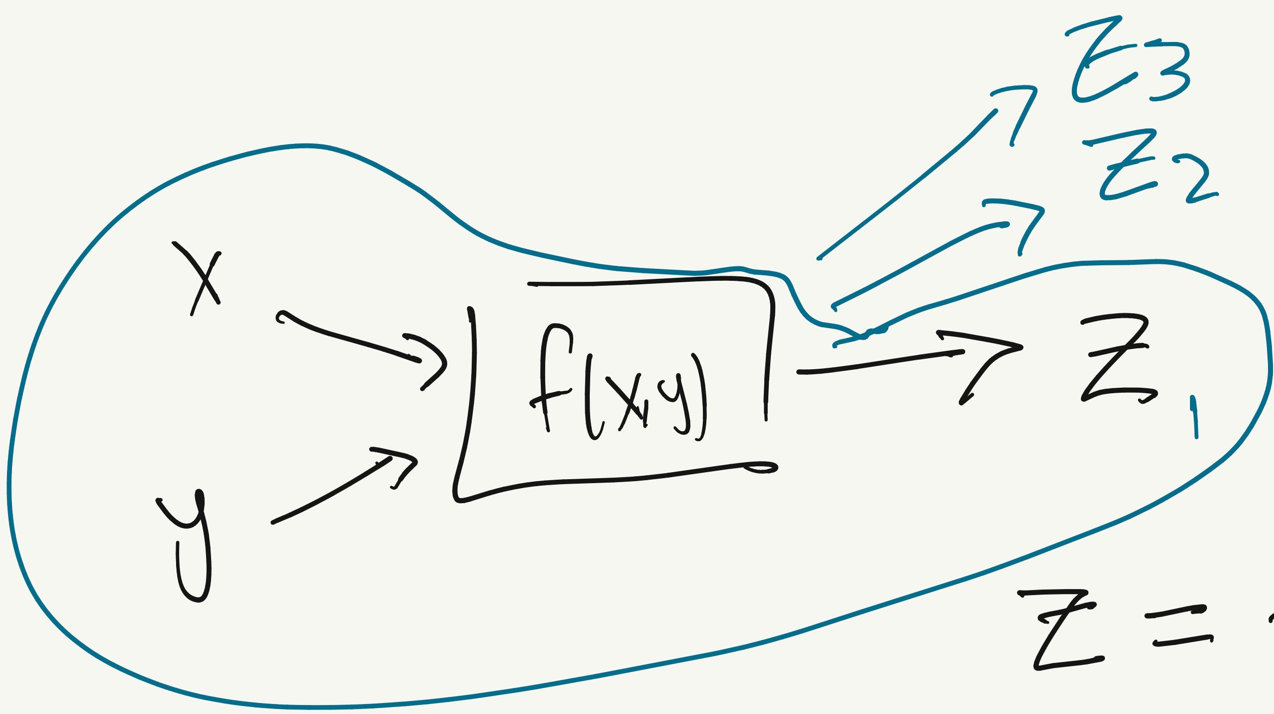
Multivariable Calculus

↑

function that has ≥ 2 inputs.



Inputs ↓

$$z = f(x,y)$$

↑ Output.

Ex

- $f(x,y) = x + y$
- n is # of sets Skis sold, p is the price of a set of Skis

$$R(n,p) = n \cdot p$$

↑ revenue function

$f(r, P, t) = P \cdot \left(1 + \frac{r}{12}\right)^{12t}$ function models monthly compound growth

__Domain__: Set of points (x, y) that we're allowed to plug-in to $f(x,y)$.

ie it's the collection of points where $f(x,y)$ is defined

__Ex__ $f(x,y) = x+y$ __Domain__: all tuples/pair (x,y) x, y real #s

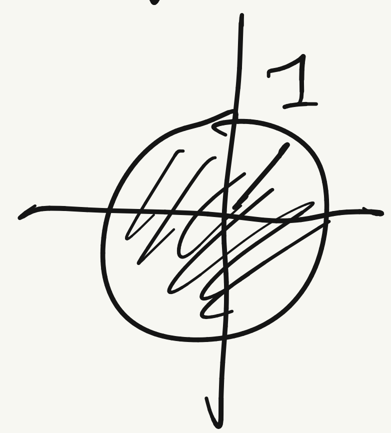
$f(x,y) = \sqrt{1 - x^2 - y^2}$

$1 - x^2 - y^2 \geq 0$

$\Downarrow$

$x^2 + y^2 \leq 1$ ← unit disk

restricted domain



Range $z = f(x,y)$ range is the collection of output values.

Ex $x+y$ → D: $\mathbb{R}^2$ ← all pairs of real #s.

R: all real #s $\mathbb{R}$ ← all real #s

$\sqrt{1-x^2-y^2}$: D: $x^2+y^2 \leq 1$

$R: [0, 1] \to 0 \le z \le 1$

$f(0,0) = \sqrt{1-0-0} = \sqrt{1} = 1$

$f(1,0) = \sqrt{1-1^2-0} = \sqrt{0} = 0.$

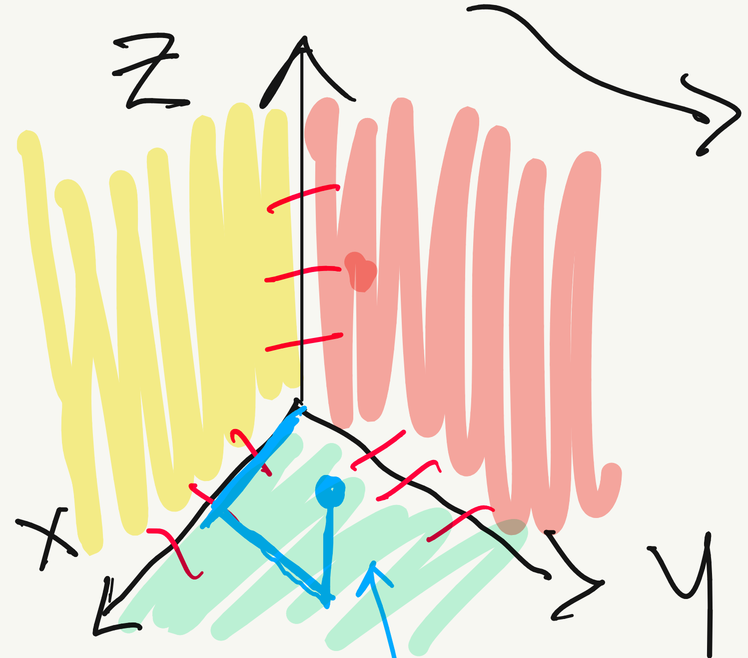
Mini intro to 3D-geometry

$\mathbb{R}^3$ = coll'n of all 3D points

Point in 3D is a triple (3-tuple) of real #s (x, y, z)

Plot a 3D point, need coord. axes.

Right-handed coord system!



(2, 3, 1)

Idx finger along +x-axis
middle finger + y-axis
thumb will point +z direction

xy-plane is the coll'n of points
(x, y, 0) ie z = 0

xz-plane
(x, 0, z)

yz-plane
(0, y, z)

Two points $P = (x_0, y_0, z_0)$

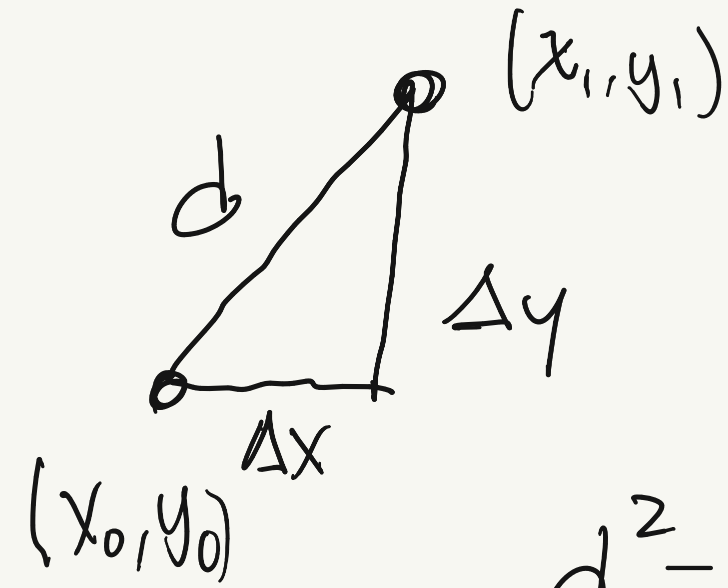
$Q = (x_1, y_1, z_1)$

distance from P to Q denoted

$$\|PQ\| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

$$= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

↑ "distance formula"



$$d^2 = \Delta x^2 + \Delta y^2$$

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

Ex Compute dist. from

① $(0,0,0)$ to $(2,1,3)$

$$d = \sqrt{(2-0)^2 + (1-0)^2 + (3-0)^2} = \sqrt{4+1+9} = \boxed{\sqrt{14}}$$

② $(0,1,2)$ to $(1,1,1)$

$$d = \sqrt{(1-0)^2 + (1-1)^2 + (1-2)^2}$$

$$= \sqrt{1^2 + 0^2 + (-1)^2} = \boxed{\sqrt{2}}$$

The eq'n of a sphere of radius R centered @ point (a, b, c) is:

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$$

Sphere = set of all points distance R away from the central point (a, b, c)

take sqrt of both sides

$$R = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$$

Note: this is distance formula!

$x^2 + y^2 + z^2 = R^2$ (0,0,0)

Sphere centered @ origin!

is the plot of the Eqn

$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$

"implicit form."