Last time:

Traces are curves of the form

$$z = f(a, y)$$

$$z = f(x, b)$$

↳ intersections of the surface $z = f(x, y)$ w/ planes $x = a$ or $y = b$ respectively

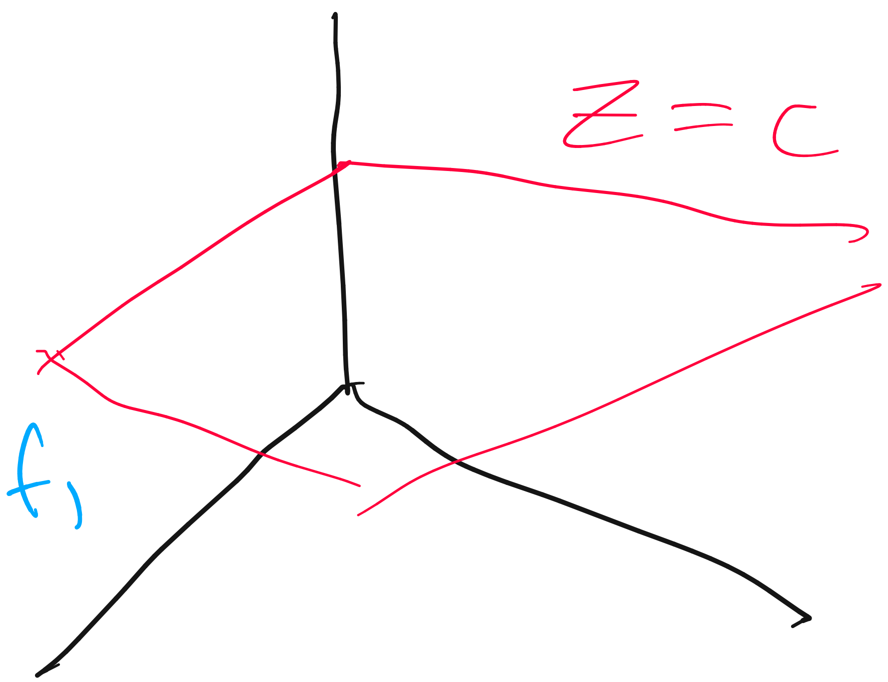
Today: Method of Contours / Level Curves

Setup: function $z = f(x,y)$

Pick a real # c & look at the set of points (x, y) such that

$$\underbrace{f(x,y) = c}$$

Curve of this form is called a contour of f, or a level curve of f.



Ex $f(x,y) = \sqrt{x^2+y^2}$

If $c < 0$ $\quad \sqrt{x^2+y^2} < 0$ $\rightarrow$ no level curves for

c negative

$c = 0$ $\quad \sqrt{x^2+y^2} = 0 \Rightarrow (x,y) = 0$

the level curve

$f(x,y) = 0$ is just a pt @

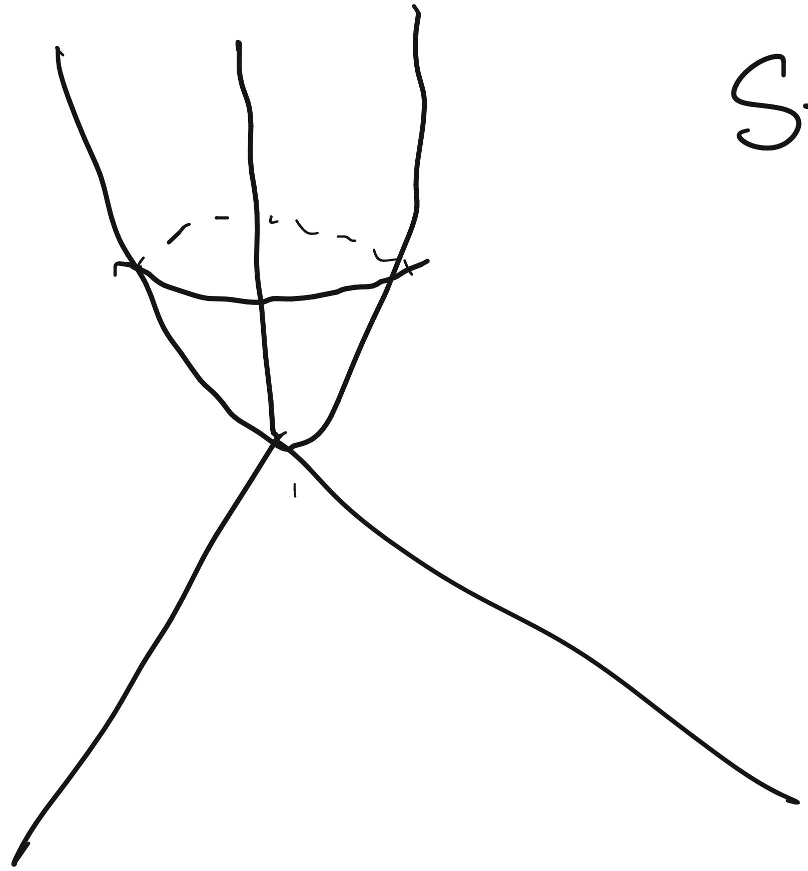
origin.

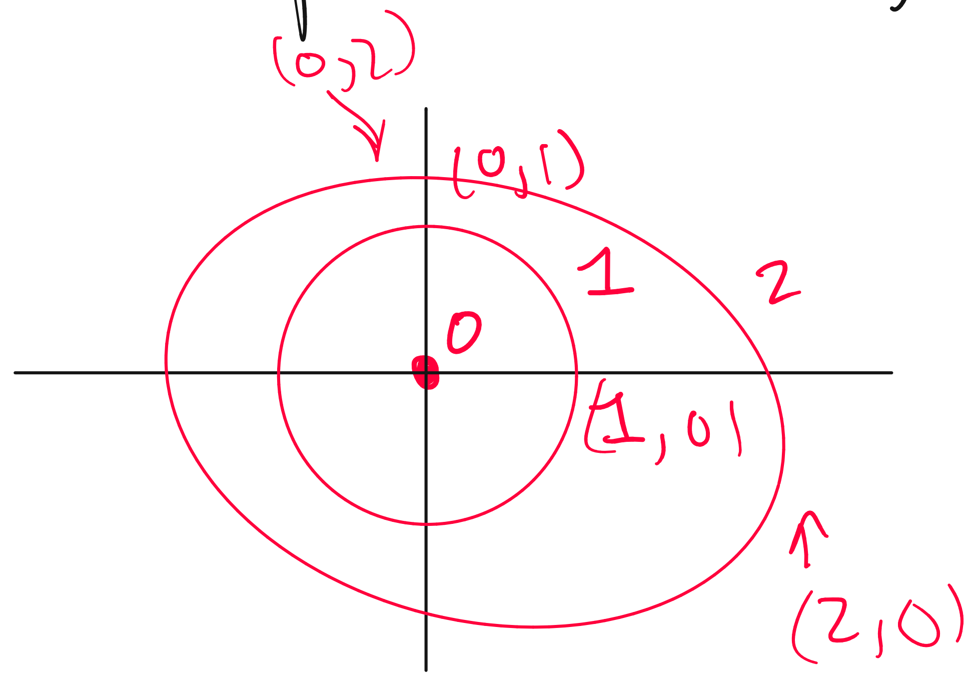
$c > 0$ $\quad \sqrt{x^2+y^2} = c \Rightarrow$

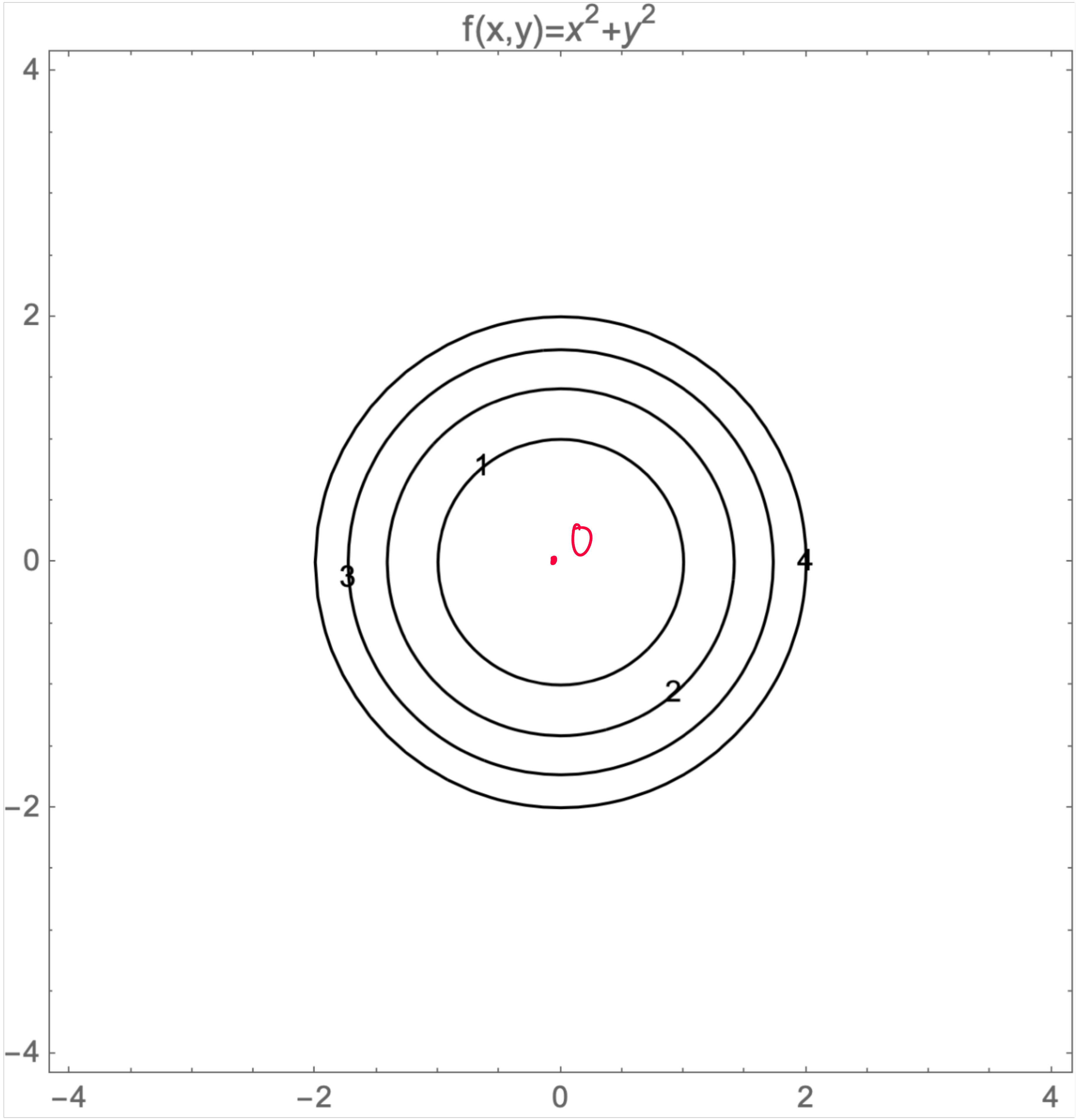
$x^2+y^2 = c^2$ $\quad$ circle of radius c ctd @

origin.

Stand @ "point" $(0, 0, \infty)$



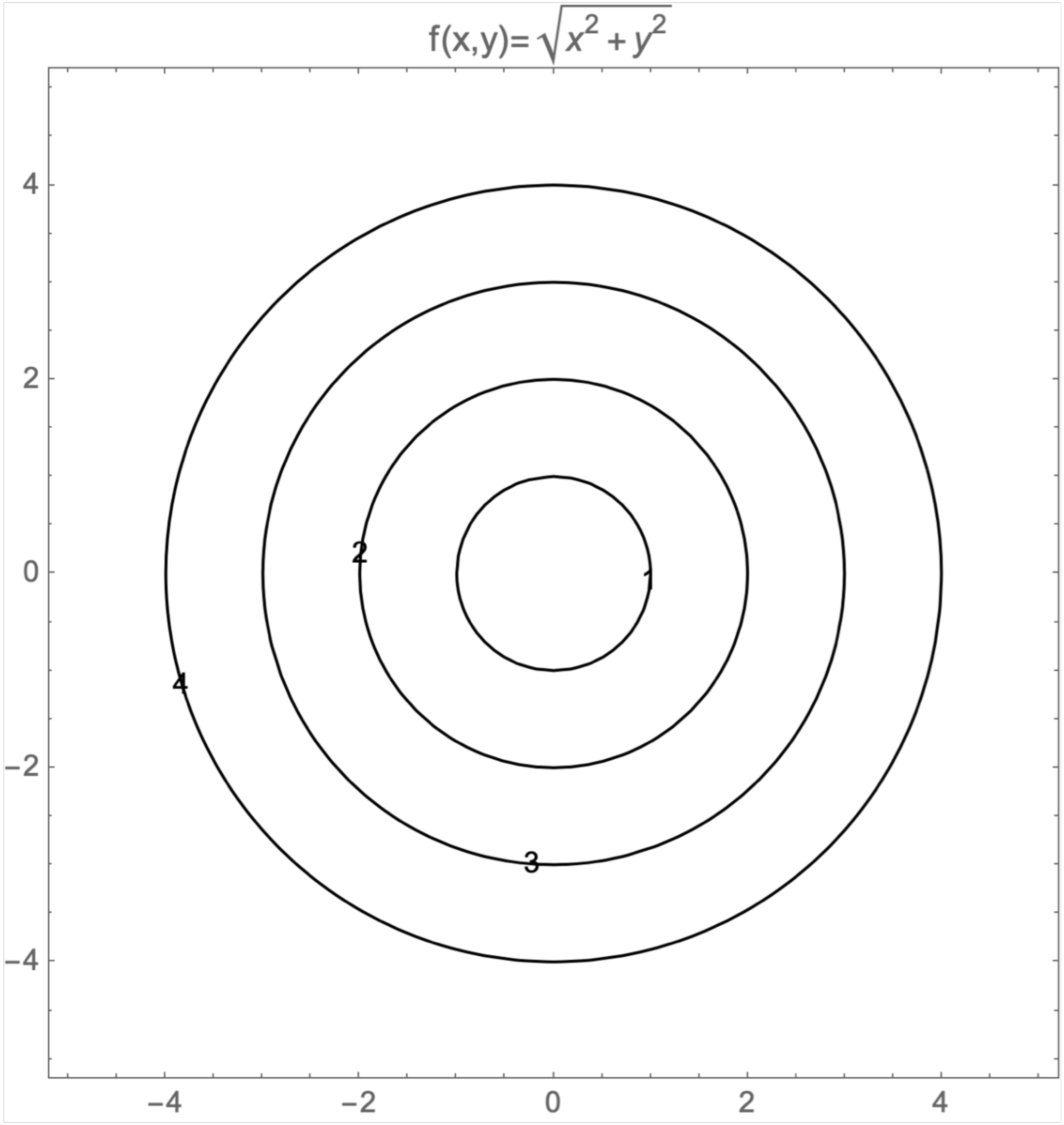


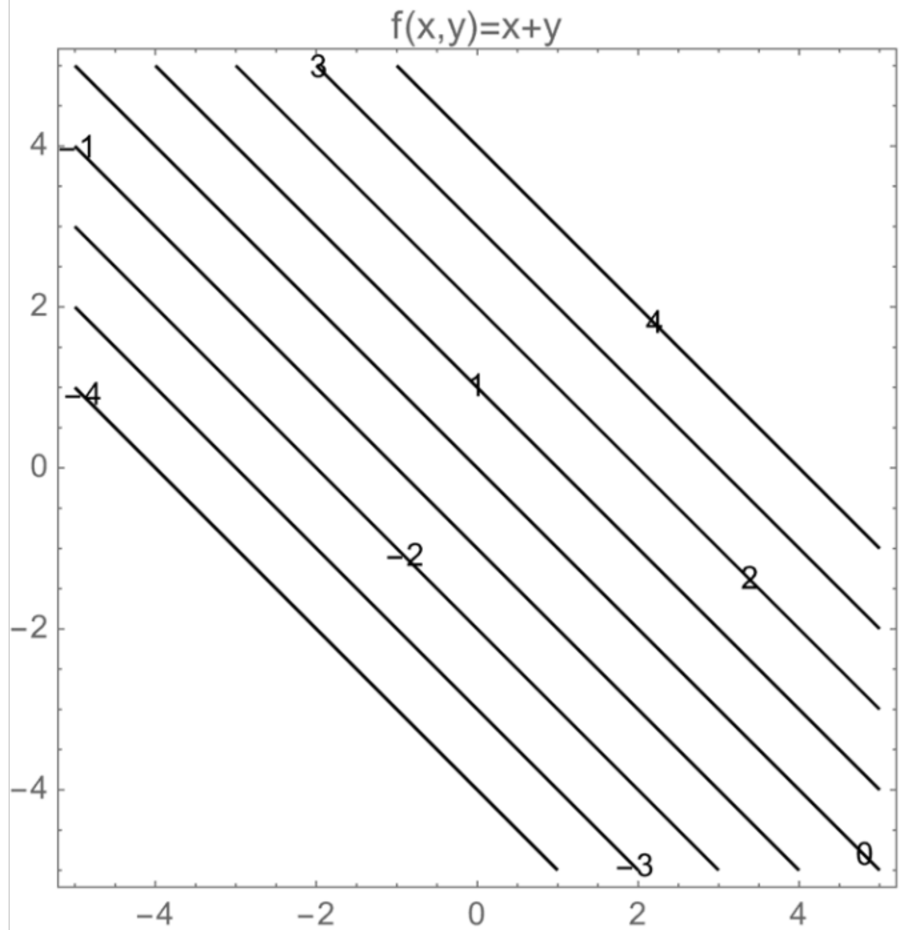


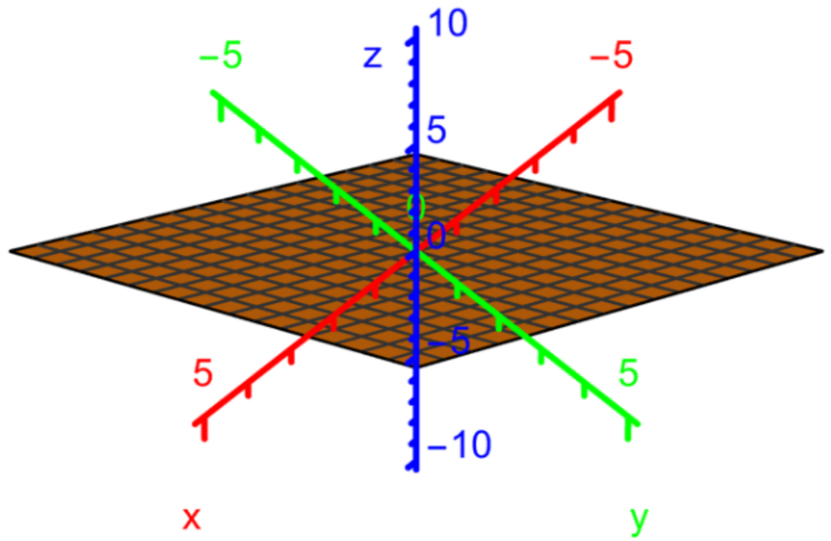
"Paraboloid of one sheet"

contours close together

$\Rightarrow$ Surface is steeper

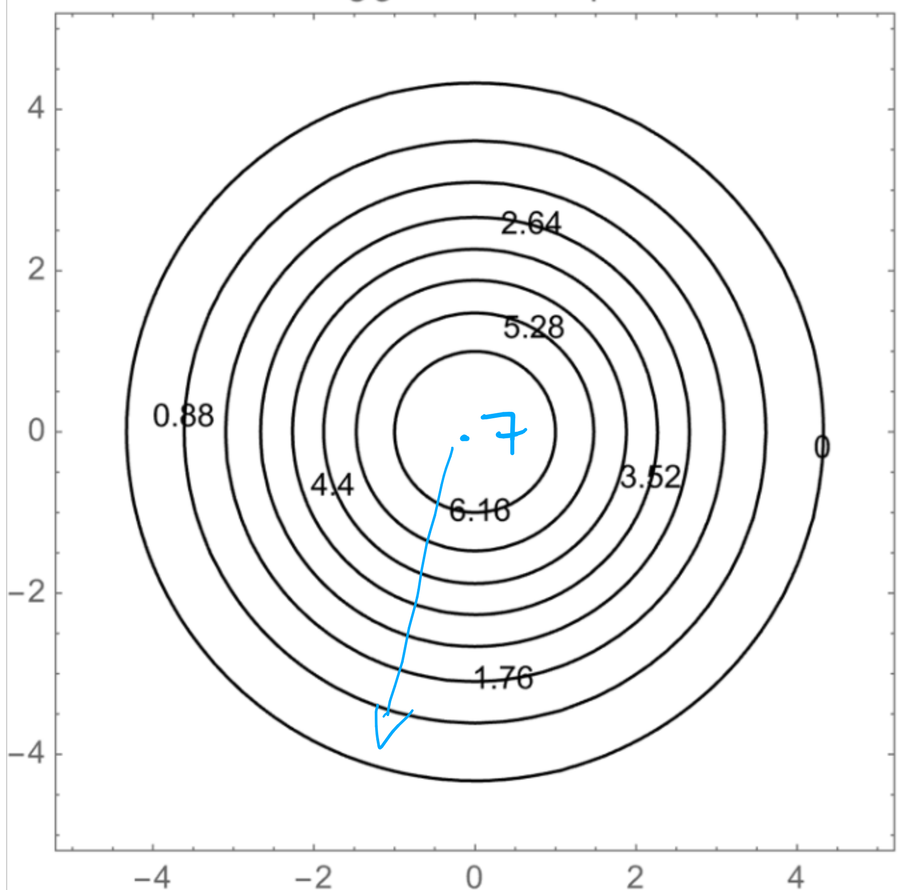

f(x,y)= $\sqrt{x^2+y^2}$
4
2
0
−2
−4
−4
−2
0
2
4
1
2
3
4

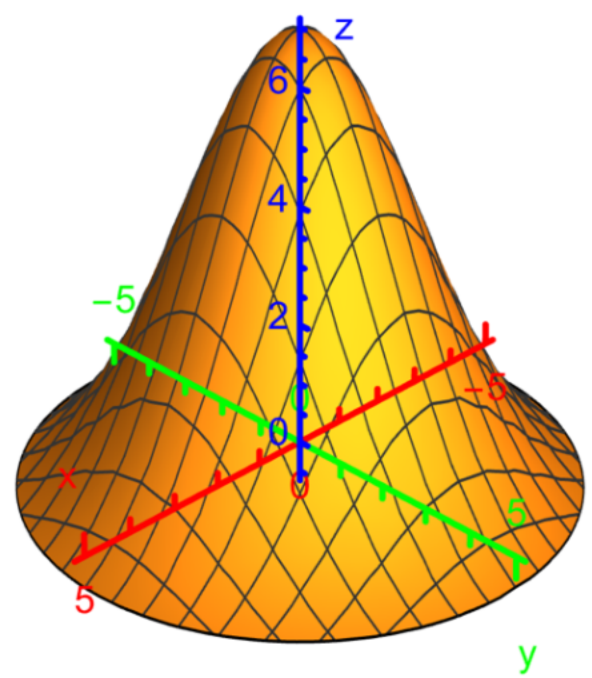




$$x+y=c \Rightarrow y=c-x$$

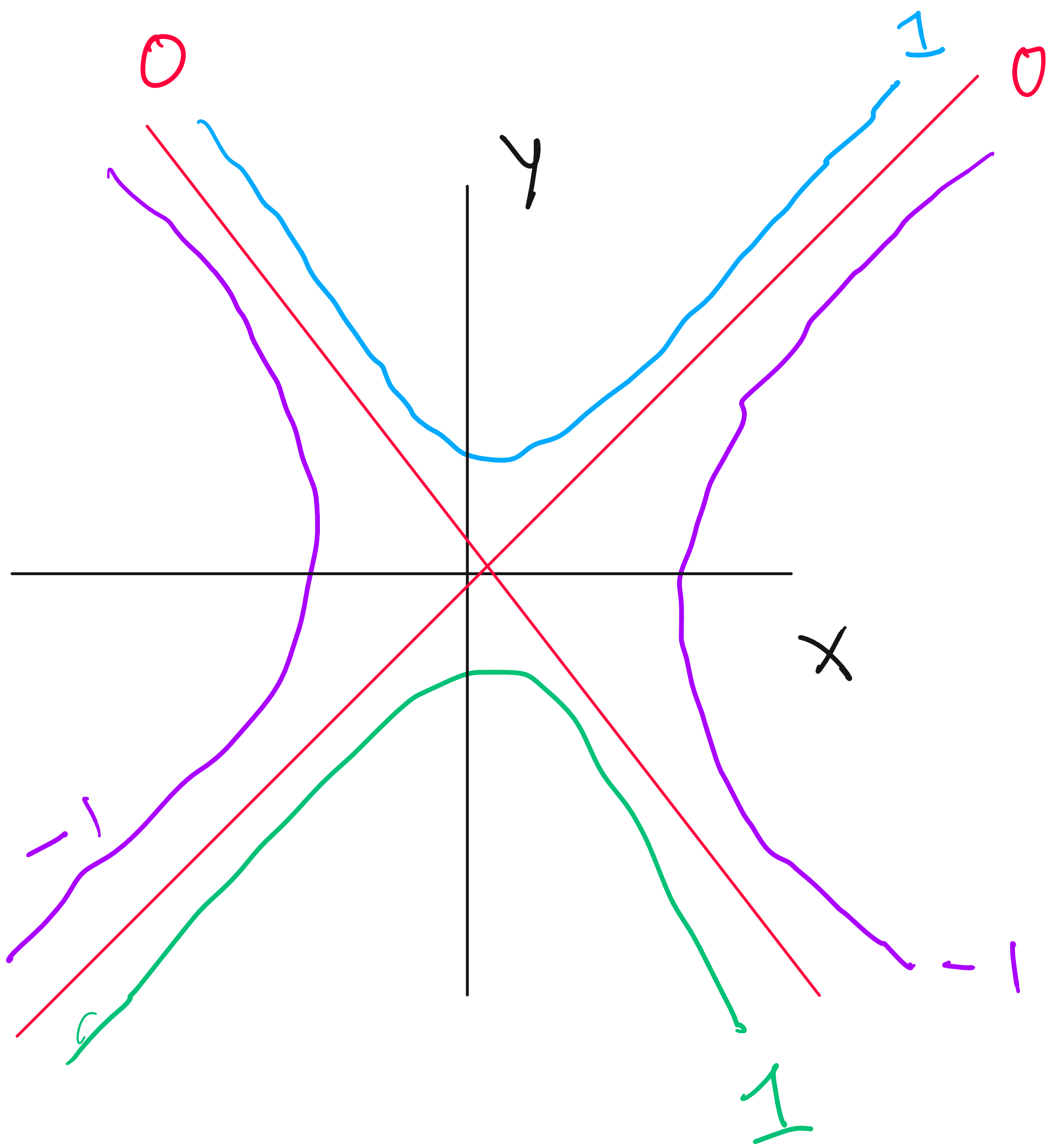
$$8\,e^{\frac{1}{9}\left(-x^2-y^2\right)}-1$$





Ex $f(x,y) = y^2 - x^2$

$C = y^2 - x^2$ ← general Eqn for a contour of <u>this</u> function.



<u>C = 0</u>

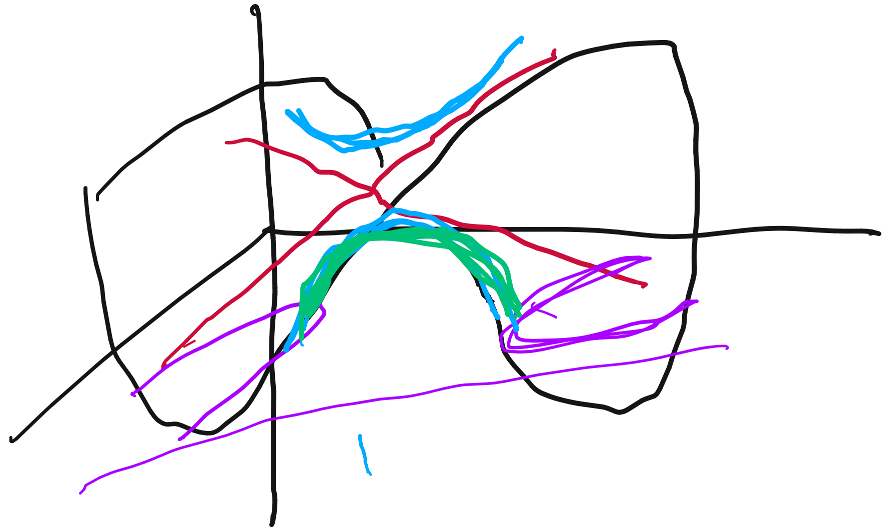
$0 = y^2 - x^2$

$\Downarrow$

$y^2 = x^2 \implies$

$y = +x$

$y = -x$



$\underline{C = 1}$

$$1 = y^2 - x^2 \Rightarrow y^2 = 1 + x^2$$

$$y = \sqrt{1+x^2}$$

$$y = -\sqrt{1+x^2}$$

$\underline{C = -1}$

$$-1 = y^2 - x^2 \rightsquigarrow y^2 = x^2 - 1 \rightsquigarrow y^2 + 1 = x^2$$

$$x = \pm\sqrt{y^2+1}$$

$z = y^3 - x(x-1)$ has 0 level curve looks similar to