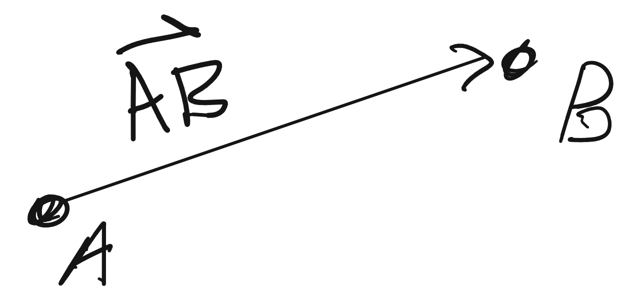
Webwork 9.1 parts 1 & 2 due tonight!

A scalar is a single real #

↳ Ex: time, pressure, temperature, or # of a things

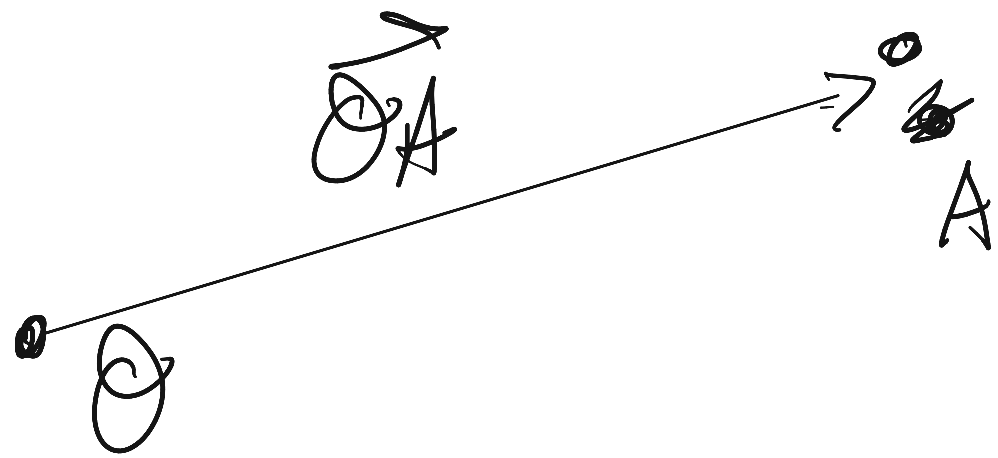
Vector: quantities that carry both magnitude & direction info.

↳ Displacement / position: tells you where you are or how to get from pt A to pt B



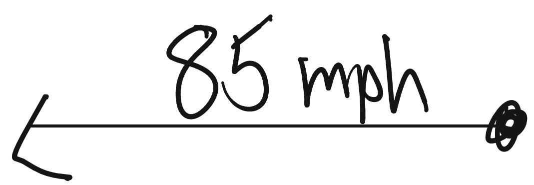
Position vector points from the origin O

to your point A.



Velocity how fast you're going & in what direction

↳ 85 mph due west



Representations of Vectors:

In 2D: Vectors are written in one of two ways

① List: $\underline{\langle a, b \rangle}$ position vector points from origin $(0,0)$ to the point (a, b)

② Unit basis vectors: $a\hat{\imath} + b\hat{\jmath}$

$\hat{\imath} = \langle 1, 0 \rangle$ $\xrightarrow{1}$

$\hat{\jmath} = \langle 0, 1 \rangle$ $\uparrow 1$

In 3D:

$$\langle a, b, c \rangle$$

or

$$a\hat{i} + b\hat{j} + c\hat{k}$$

$\hat{k}$ points in $+z$ dir.

$\hat{k} = \langle 0, 0, 1 \rangle$

Given two objects of the same type

When are they Equal?

$\langle u_1, u_2, \ldots, u_n \rangle = \vec{u}$

$\langle v_1, \ldots, v_n \rangle = \vec{v}$

$\vec{u} = \vec{v}$ if & only if $u_i = v_i$ for each i

"Equal vectors have the same components"

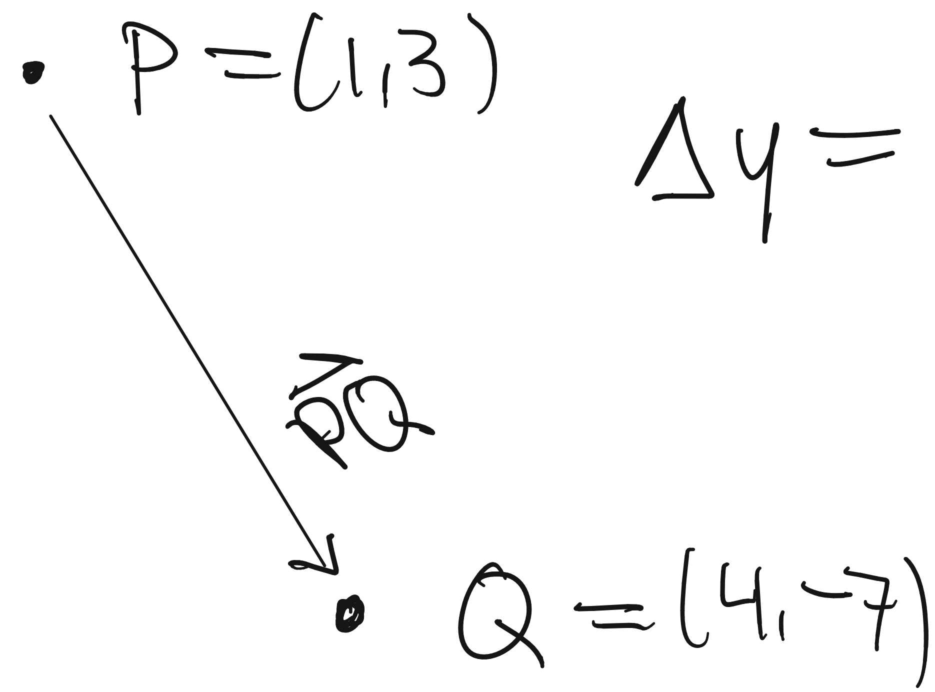
"slot" of a vector.

Ex (Finding a displacement vector)

$P = (1,3)$, $Q = (4,-7)$

Find the disp. vector $\overrightarrow{PQ}$

$\Delta x = 3$



$\Delta y = -10$

Formula:

$\overrightarrow{PQ} =$ "final" $-$ "initial"

$(4,-7) - (1,3)$

$= \boxed{\langle 3, -10 \rangle}$

$$P = (1, 2, 3)$$

$$Q = (4, -5, 6)$$

$$\vec{PQ} = Q - P = \langle 3, -7, 3 \rangle$$

Using Vectors:

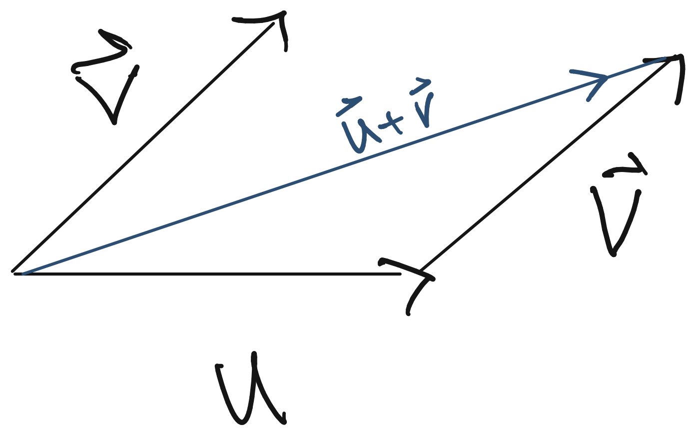
Setup: a, b, λ real #s,

$\vec{u}, \vec{v}$ be n-dim'l vectors

① Addition $\vec{u}+\vec{v} = \langle u_1, \cdots, u_n \rangle + \langle v_1, \cdots, v_n \rangle$

$$= \langle u_1+v_1, u_2+v_2, \cdots, u_n+v_n \rangle$$

in general $\vec{u}+\vec{v} = \vec{v}+\vec{u}$



"Tip-to-tail" or parallelogram method.

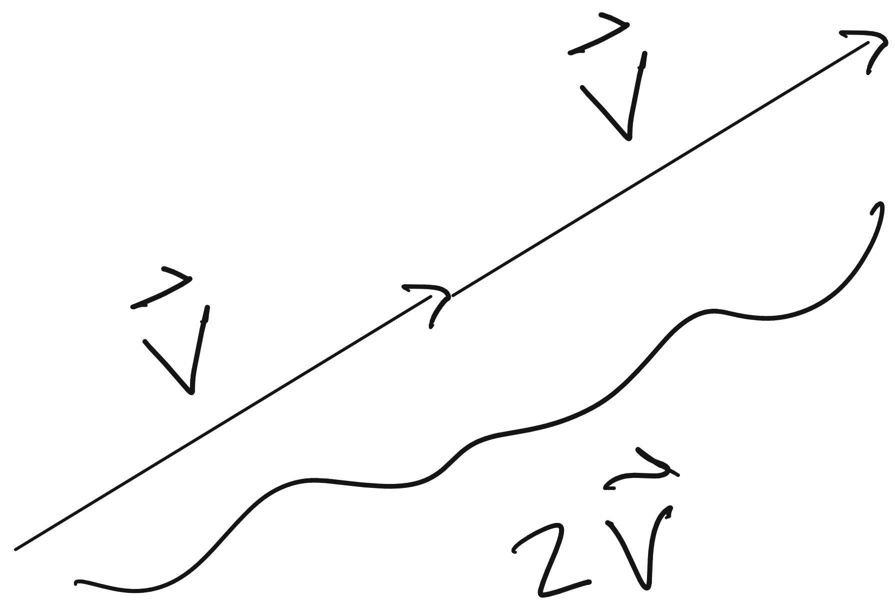
$$\vec{0} = \langle 0, 0, \dots, 0 \rangle$$

Zero vector.

$$\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$$

② Scalar multiplication

$$\lambda \vec{v} = \lambda \langle v_1, \dots, v_n \rangle = \langle \lambda v_1, \dots, \lambda v_n \rangle$$



__note:__ $\lambda \vec{v}$ is parallel to $\vec{v}$.

if $\lambda < 0$:

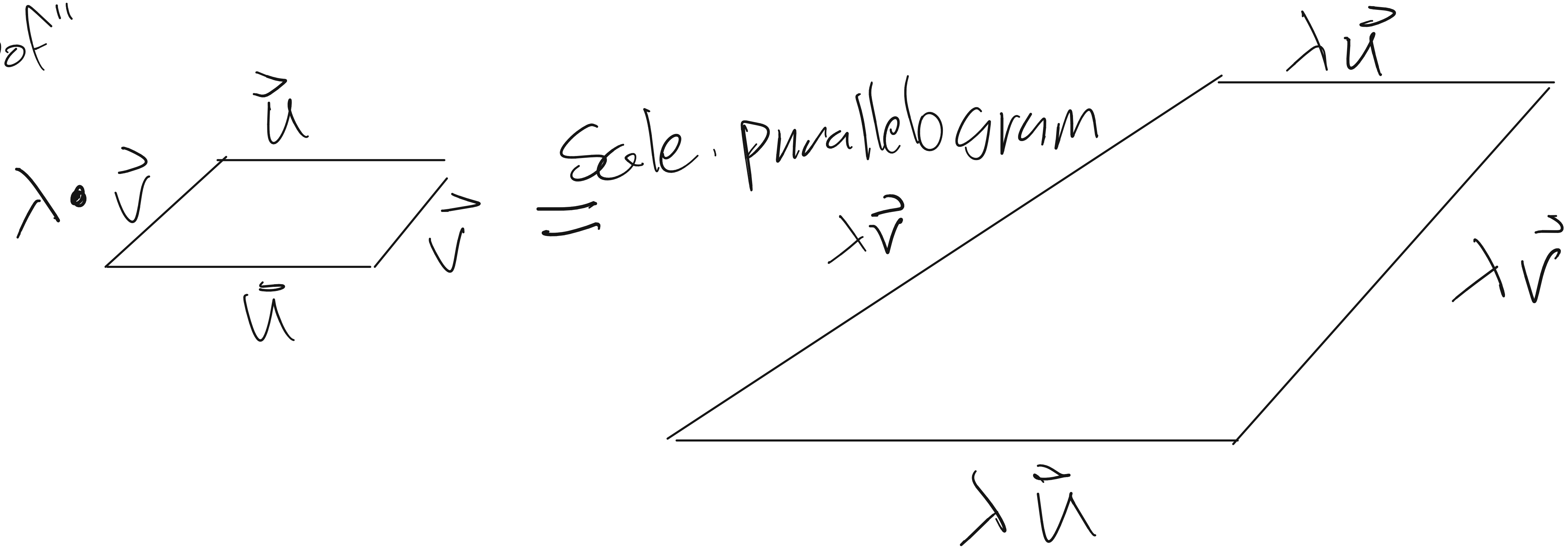
Same line, diff. directions.

③ Distributive rule:

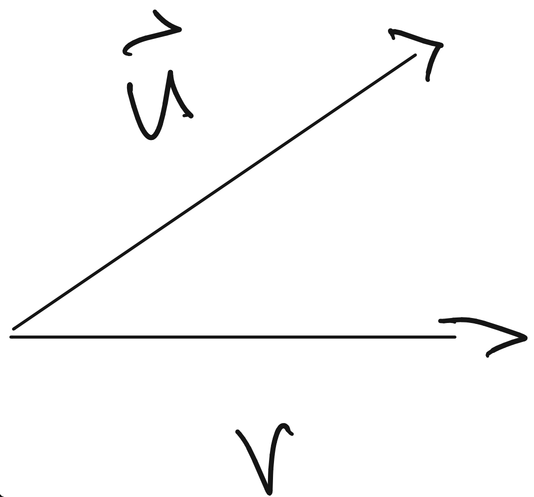
$$\lambda(\vec{u}+\vec{v}) = \lambda\vec{u} + \lambda\vec{v}$$

Scalars distribute over vector addition.

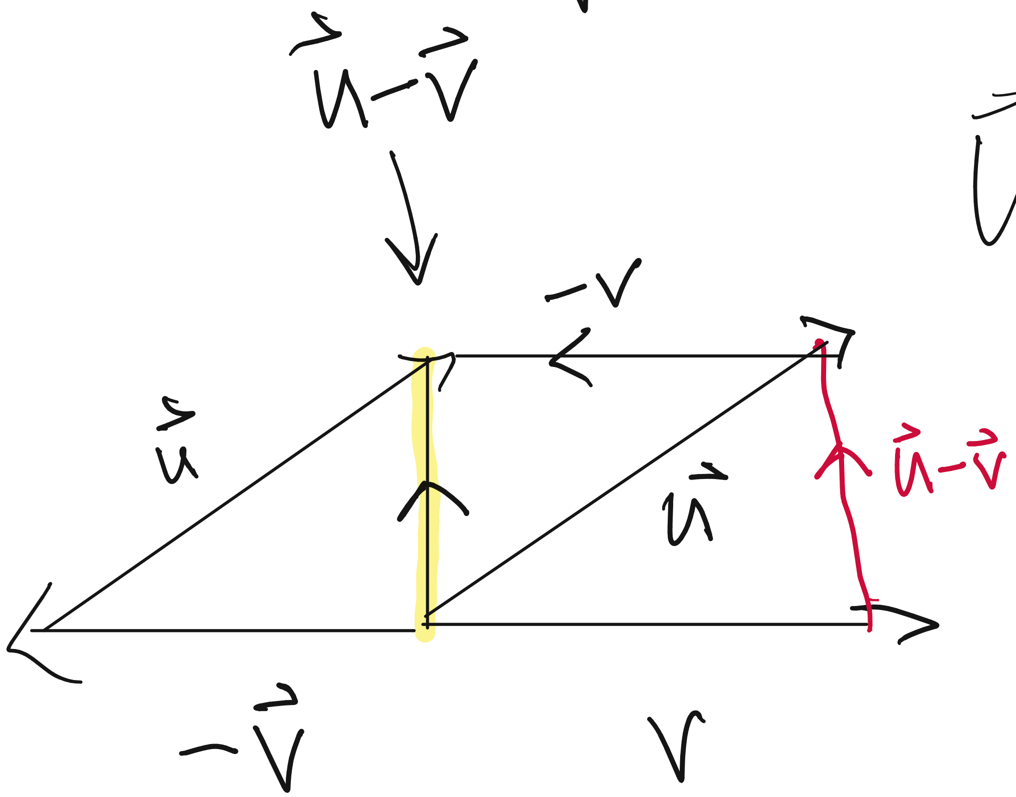
"proof"



rules 1 & 2 tell us how to do <u>Subtraction</u>:



$$\vec{u} - \vec{v} = \vec{u} + (-1\ \vec{v})$$



$\vec{u} - \vec{v}$ points from head of $\vec{v}$ to head of $\vec{u}$

Norm of a vector aka magnitude or length

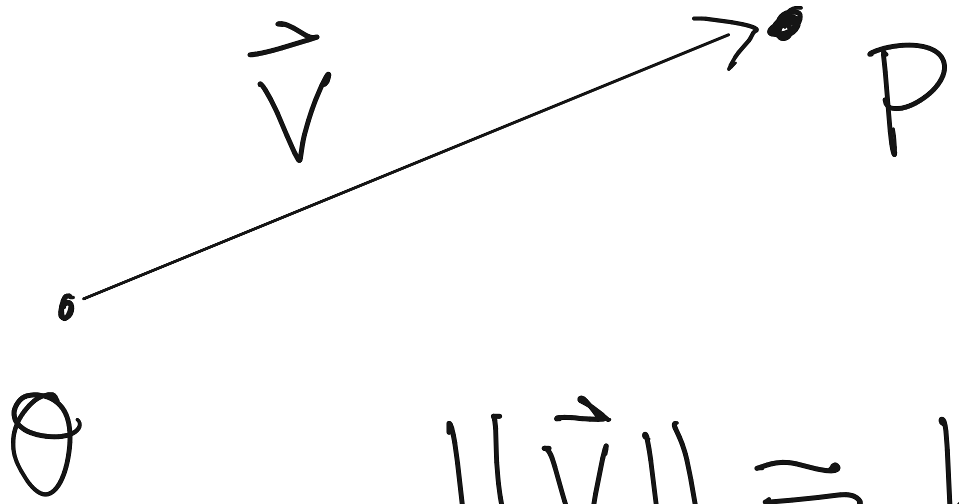
$\vec{v} = \langle v_1, \dots, v_n \rangle$

$$\|\vec{v}\| = \left(\sum_{i=1}^{n} v_i^2\right)^{1/2} = \sqrt{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}$$

$\vec{v}$ displacement vector from the origin $O = (0, 0, \dots 0)$

to $P = (v_1, \dots, v_n)$

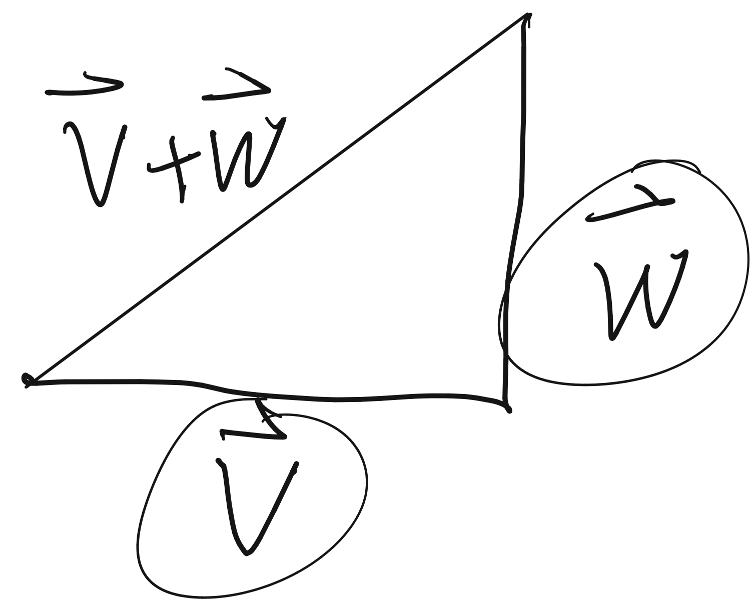
& $\|\vec{v}\|$ is length of line segment from O to P.



$$\|\vec{v}\| = \text{length}(\overrightarrow{OP})$$

Triangle inequality

$\vec{v}, \vec{w}$ two n-dim'l vectors



$$\|\vec{v}+\vec{w}\| \le \|\vec{v}\| + \|\vec{w}\|$$

Unit Vectors: a vector $\vec{w}$ is a unit vector

if $\|\vec{w}\| = 1$.

Notation: the hat $\hat{w}$ means that $\hat{w}$ is a unit vector.

Method: if $\vec{w} \neq \vec{0}$ then we can form a unit vector along $\vec{w}$ by setting

$$\boxed{\hat{w} = \frac{\vec{w}}{\|\vec{w}\|}}$$