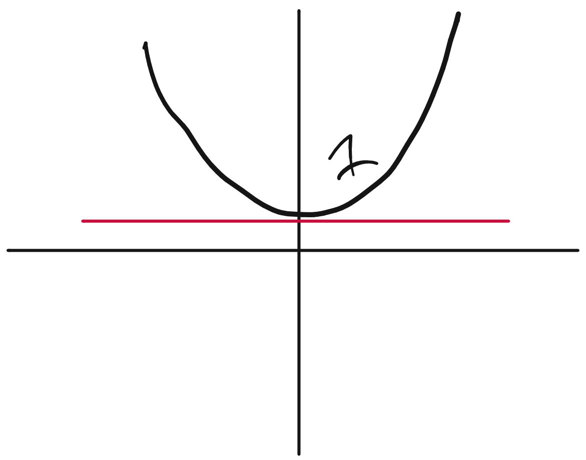
Review: A slice of a function is a curve of the form

$z = f(x, b)$ or $z = f(a, y)$ for some real #s a, b.

Big Idea: Set one variable = constant & look how the func. changes with resp. to the other!

Ex $z = x^2y + y^4$. look @ $y = 1$ trace (= slice)

$f(x,1) = x^2 \cdot 1 + 1^4 = x^2 + 1$



Q: What is $\left.\dfrac{df(x,1)}{dx}\right|_{x=0}$?

A! $\left.\dfrac{df(x,1)}{dx}\right|_{x=0} = 2x\Big|_{x=0} = 0$

Pretend one variable is a constant & take a derivative!

Some sources may use Δx or Δy in place of h

Partial derivative:

What we just did (w/o actually doing a limit) was compute the limit:

$$\lim_{h \to 0} \frac{f(0+h,1) - f(0,1)}{h} = f_x(0,1)$$

don't:

$$\cancel{\frac{df}{\partial x}(0,1)}$$

$$= \frac{\partial f}{\partial x}(0,1)$$

Def'n: let $f(x,y)$ a function, let (a,b) be a point @ which $f(x,y)$ is continuous.

Then: $$f_x(a,b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$$

$$f_y(a,b) = \lim_{h \to 0} \frac{f(a, b+h) - f(a,b)}{h}$$

Ex $\quad f(x,y) = \dfrac{xy^2}{x+1} \quad$ Point is $(a,b) = (1,2)$

Compute $f_x(1,2) = \lim\limits_{h \to 0} \dfrac{f(1+h, 2) - f(1,2)}{h}$

$$\frac{1 \cdot 2^2}{1+1} = \frac{4}{2}$$

$$= \lim_{h \to 0} \frac{\frac{(1+h) \cdot 4}{2+h} - 2\,\frac{2+h}{2+h}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4+4h}{2+h} - \frac{4+2h}{2+h}}{h} = \lim_{h \to 0} \frac{2\cancel{h}}{2\cancel{h}+h^{\cancel{2}}}$$

$$\frac{1}{h} \cdot \left[\sim \right]$$

$$= \lim_{h \to 0} \frac{2}{2+h} = 1$$

$$f_x(1,2) = 1$$

$$f(x,y) = \frac{xy^2}{x+1} \quad \text{Point is } (a,b) = \underline{(1,2)}$$

$$f_y(1,2) = \lim_{h \to 0} \frac{f(1,2+h) - f(1,2)}{h}$$

$$\frac{1 \cdot 2^2}{1+1} = \frac{1 \cdot 4}{2}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2}(2+h)^2 - \boxed{2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2}[4+2h+h^2] - 2}{h}$$

$$= \lim_{h \to 0} \frac{2 + h + \frac{1}{2}h^2 - 2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h + \frac{1}{2}h^2}}{\cancel{h}} = \lim_{h \to 0} \left(1 + \frac{1}{2}h\right) = 1$$

$$f_y(1,2) = 1$$

Ex We can do this using our Calc I skills!

↳ When computing $\frac{\partial f}{\partial x}$ treat any y you see as though it were a constant.

no y.

Ex $f(x,y) = 3x^2 - 2x^2y^5$

$$\frac{\partial f}{\partial x} = 6x - 4xy^5$$

$$f_y = 0 - 2x^2 \cdot 5y^4 = -10x^2y^4$$

Ex $f(x,y) = \arctan(xy)$

Remember $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$

$\arctan(2x)$

$\arctan(2y)$

$$\frac{\partial f}{\partial x} = y \cdot \arctan'(xy)$$

$$= \frac{y}{1+(xy)^2}$$

Chain rule!

$$\frac{\partial f}{\partial y} = x \cdot \arctan'(xy) = \frac{x}{1+(xy)^2}$$

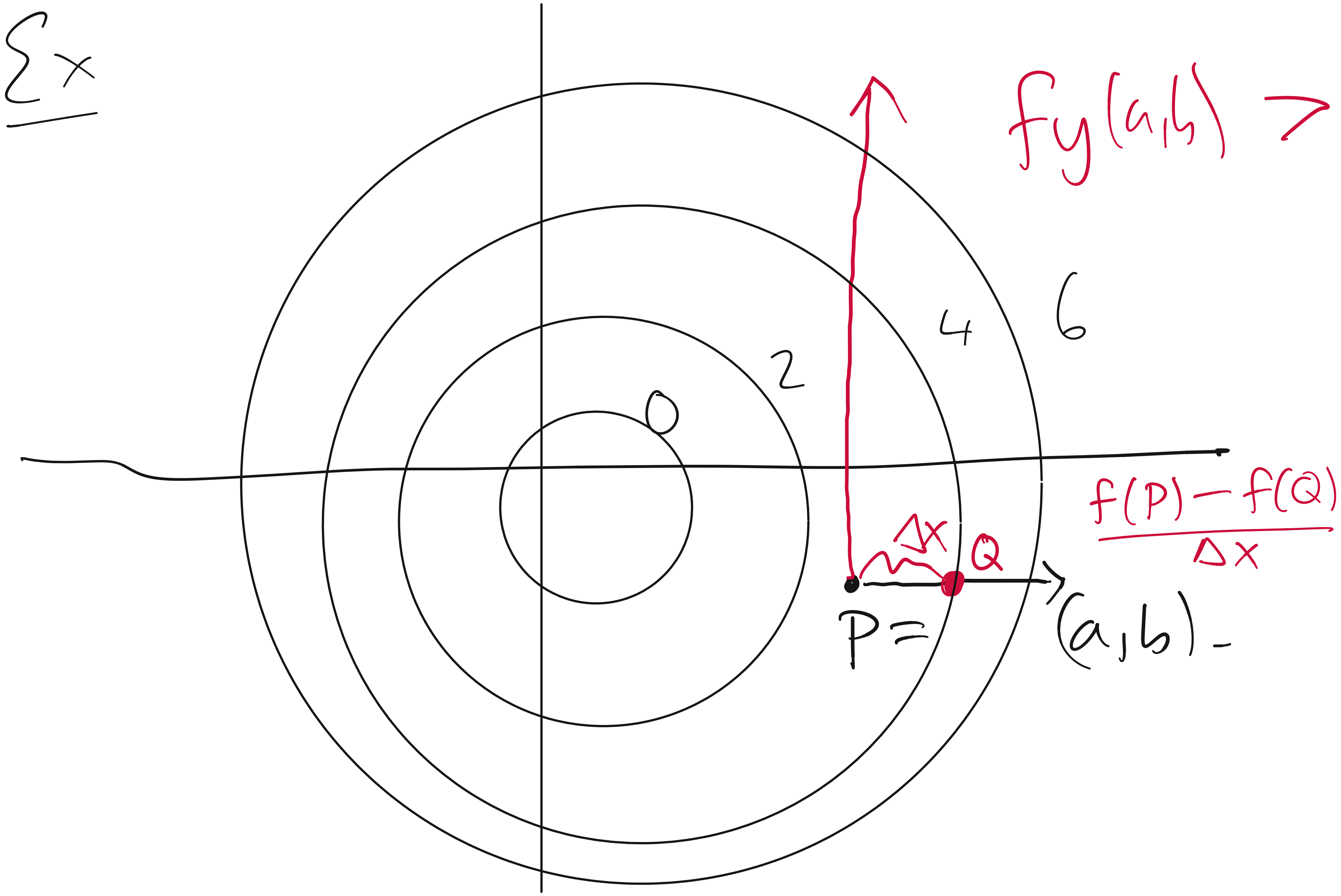
Interpretation of Partials:

If $f_x(a,b) > 0$ then f is <u>increasing</u> in $+x$ direction @ (a,b)

$f_x(a,b) < 0$ then f is <u>decr.</u> in the $+x$-direction @ (a,b)

& similarly for $f_y(a,b)$

Ex



$f_y(a,b) > 0$ b/c we're increasing!

Q: is $f_x(a,b)$ +'ve, −'ve or 0 ?

b/c as we walk in the +x dir. from P, our z values incr.

$y \downarrow$ \ $x \rightarrow$	-5	0	5
10	8	15	0
5	-7	-10	-15
0	-12	-17	-22

$$\frac{-10-(-7)}{0-(-5)} = \boxed{\frac{-3}{5}}$$

Avg these two estimates to get a better estimate.

$\Delta z / \Delta y$

$\frac{\Delta z}{\Delta x}$

Estimate $f_x(0,5)$, $\approx \frac{f(0+h,5)-f(0,5)}{h}$

$h=5$ here

$$= \frac{-15-(-10)}{5-0} = \frac{-5}{5} = \boxed{-1}$$

$f_y(0,5)$. $\boxed{-4/5}$