Last time: Partial derivatives:

↳ only took 1 deriv. @ a time

$f = f(x,y)$

$\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ ← is f incr. in y dir. or not?

↑

is f incr. in x dir or not?

Today: Second-order Partials:

$$f_{xx} = (f_x)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

is the second order partial deriv. of $f(x,y)$ with respect to x, twice!

Unmixed partials

Derivative of derivative!

$$f_{yy} = (f_y)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

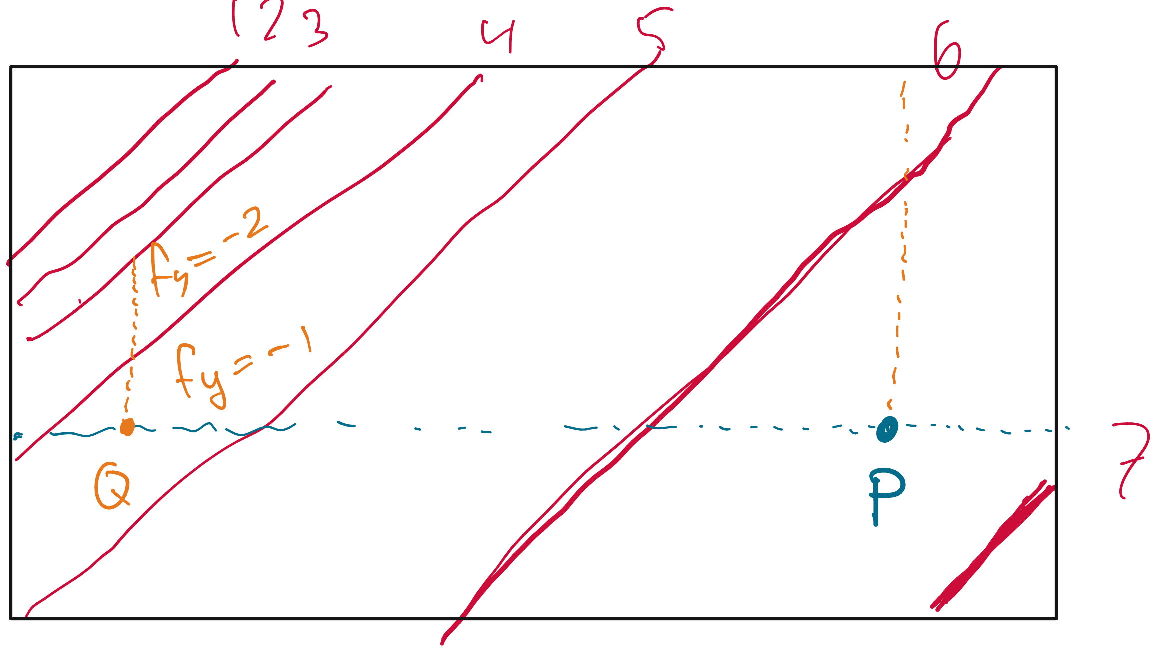
is 2^{nd} order partial deriv. of $f(x,y)$ w.r.t. y twice.

$2^{\underline{nd}}$ order partials tell us about concavity of $f(x,y)$.

f_{xx} tells us if f is concave up/down (in x-direction)

f_{yy} tells us if f is concave ↑/↓ (in y-dir.)

↳ on a contour diagram:



f_{xx} tells us about density of the contours in x-dir.

here, $f_{xx} < 0$ b/c contours get less dense in +x dir. and

b/c $f_x > 0$.

$f_y < 0$ b/c f is decr.ing

f_{yy} tells us how f_y changes as we walk in +y dir.

$f_{yy} < 0$ b/c as we walk in +y-dir, our f_y values decrease.

If f_s and f_{ss} have same sign,

then density of your contours is increasing.

if f_s, f_{ss} different signs, contours get less dense.

(in +s-direction)

Ex $f(x,y) = \underline{2x^3y} - 4y^2x$

$f_x(1,1) = 6-4 = 2$

f_{xx}, f_{yy}

partial deriv. wrt x.

look @ point (1,1)

$f_{xx} = (f_x)_x = (6x^2y - 4y^2)_x$

$= \boxed{12xy}$

$f_{xx}(1,1) = 12$

$\rightarrow$ f is CC↑ @ (1,1) in x-dir.

$$f_{yy} = (f_y)_y = (\underbrace{2x^3}_{0} - \underbrace{8xy}_{-8x})_y$$

$$= \boxed{-8x}$$

$f_{yy} = -8$ @ $(1,1)$.

So f is CC↓ in y-dir. @ $(1,1)$.

Mixed Partials:

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}.$$

first ~~take~~ x-deriv, then take y-deriv.

↳ tells us how f_x changes in +y-direction

$$f_{yx} = (f_y)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

↳ first y-deriv then x-deriv.

↳ tells us how f_y changes in + x-direction.

$f(x,y) = \underline{2x^3y} - 4y^2x$

$f_x = 6x^2y - 4y^2$

@ $(1,1)$

$f_x = 2.$ Incr. in x-dr.

$f_y = 2x^3 - 8yx$

@ $(1,1)$

$f_y = -6$ decr. in y dir.

$f_{xy} = (f_x)_y = 6x^2 - 8y$

$f_{yx} = (f_y)_x = 6x^2 - 8y$

$\boxed{\text{! Same !}}$

Theorem Clairaut's Theorem

If $f(x,y)$ is a func. of two variables st

f_{xy} and f_{yx} are continuous functions @ (a,b),

then $f_{xy}(a,b) = f_{yx}(a,b)$

Rmk: this says that the order of mixed partials in general doesn't matter if the derivatives are continuous.

Interpretations of f_{xy}, f_{yx}

tells me if f_x is incr/decr. in y-dir.

& ~'ly for f_{yx}.

Another Interpretation

f_{xy} (or f_{yx}) tells us how f "twists".