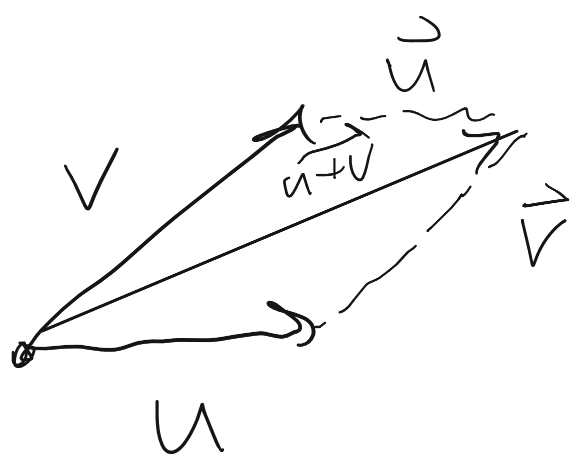
Exam Review:

Exam is over § 9.1 – 9.5, 10.1 – 10.7.



$\vec{u}+\vec{v}$

Explan tan, sin.

$\vec{u}, \vec{v}$

$$\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos(\theta)$$

Solve ↳ $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|u\|\cdot\|v\|}$

$$\theta = \cos^{-1}(\text{''} \quad \text{''})$$

$\vec{u} \cdot \vec{v} = 4$

This was a §9.4 WW Problem

$\|\vec{u} \times \vec{v}\| = 3$

find $\tan(\theta)$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos\theta$$

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \cdot \|\vec{v}\| \sin\theta$$

$$\frac{\|\vec{u} \times \vec{v}\|}{\vec{u} \cdot \vec{v}} = \frac{\cancel{\|\vec{u}\|} \cdot \cancel{\|\vec{v}\|} \sin\theta}{\cancel{\|\vec{u}\|} \cdot \cancel{\|\vec{v}\|} \cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{3}{4}$$

3 point method for finding Eqn of a plane

P, Q, R don't lie on the same line
(non-colinear points)

① find disp. vectors $\vec{PQ}, \vec{PR}$

② $\vec{n} = \vec{PQ} \times \vec{PR}$

$\vec{n} = \langle a, b, c \rangle$

$P = (x_0, y_0, z_0)$

③ normal vector, use P as your "base point"

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0.$$

$$f(x,y) = \frac{x-y}{x+y} \quad @ \ (0,0)$$

two paths: ① $y = x$

$$\lim_{\substack{x \to 0 \\ y = x}} f(x,y) = \lim_{x \to 0} f(x,x) = \lim_{x \to 0} \frac{x-x}{x+x}$$

$$= \lim_{x \to 0} \frac{0}{2x} = \boxed{0}$$

② $y = 2x$

$$f(x,2x) = \frac{x-2x}{x+2x} = \frac{-x}{3x} \qquad \lim_{x \to 0} \frac{-x}{3x} = -\frac{1}{3} \neq 0.$$

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

Most Complicated Ex: deg 2 poly in x, y.

$$7x^2 + xy + y^2 + 1$$

Tangent plane $f(x,y)$ a point $P=(a,b)$

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b).$$

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if $y=f(x)$ (Calc 1)

$$T = f(x_0) + f'(x_0)(x-x_0)$$

$P = (a,b)$ is a critical point of $f(x,y)$.

$\vec{\nabla} f(p) = \vec{0}$

$D = f_{xx}(p) \cdot f_{yy}(p) - f_{xy}(p)^2$

- $D > 0$, $f_{xx} > 0$ or $f_{yy} > 0 \rightarrow$ p is local min
- $D > 0$, f_{xx} or $f_{yy} < 0$, $\rightarrow$ p local max
- $D < 0$, $\rightarrow$ Saddle point
- $D = 0$, test is inconclusive.

3 visualizations: · Surface plots (3D)
· Contour plots
· Slices

Exam 1 Outline (Motivating Questions)

9.1: Functions of several variables and 3-Dimensional Space

- What is a function of several variables? What do we mean by the domain of a function of several variables?
- How do we find the distance between two points in $\mathbb{R}^3$?
- What is the equation of a sphere in $\mathbb{R}^3$? $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$
- What is a trace of a function of two variables? What does a trace tell us about a function? = Slice
- What is a level curve of a function of two variables? What does a level curve tell us about a function? = Contour.

9.2: Vectors

- What is a vector? (notations for vectors → $\langle , , \rangle$; $\hat{i}\hat{j}\hat{k}$ not'n)
- What does it mean for two vectors to be equal?
- How do we add two vectors together and multiply a vector by a scalar?
- How do we determine the magnitude of a vector? What is a unit vector, and how do we find a unit vector in the direction of a given vector?

= norm = length

9.3: Dot Product

- How is the dot product of two vectors defined and what geometric information does it tell us?
- How can we tell if two vectors in $\mathbb{R}^n$ are perpendicular?
- How do we find the projection of one vector onto another?

9.4: Cross Product

Righthand rule.

- How and when is the cross product of two vectors defined?
- What geometric information does the cross product provide?

9.5: Lines and planes in space

- How are lines in $\mathbb{R}^3$ similar to and different from lines in $\mathbb{R}^2$?
- What is the role that vectors play in representing equations of lines, particularly in $\mathbb{R}^3$? $\vec{OP}+t\vec{v}=\vec{r}(t)$
- How can we think of a plane as a set of points determined by a point and a vector? Eq'n of plane: $\vec{n}\cdot(\vec{P}-\vec{P_0})=0$
- How do we find the equation of a plane through three given non-collinear points?

$\vec{n}$ is normal vector

10.1: Limits

- What do we mean by the limit of a function $f(x, y)$ of two variables at a point (a, b)
- What techniques can we use to show that a function of two variables does not have a limit at a point (a, b)

Idea: Show that the limit along two diff paths disagree.

- What does it mean for a function $f(x, y)$ of two variables to be continuous at a point?

→ 10.2: First-order partials

- How are the first-order partial derivatives of a function $f(x, y)$ of the independent variables x and y defined?
- Given a function f of the independent variables x and y, what do the first-order partial derivatives f_x and f_y tell us about f?

10.3: Second-order partials Clairaut's Theorem: $f_{xy} = f_{yx}$ if f_{xy}, f_{yx} are cts

- Given a function f of two independent variables x and y, how are the second-order partial derivatives of f defined?
- What do the second-order partial derivatives f_{xx}, f_{yy}, f_{xy}, and f_{yx} of a function f tell us about the function's behavior?

10.4: Linearization: Tangent plane and differentials

- What does it mean for a function of two variables to be locally linear at a point?
- How do we find the equation of the plane tangent to a locally linear function at a point?
- What is the differential of a multivariable function of two variables and what are its uses?

10.5: Chain Rule

- What is the Chain Rule and how do we use it to find a derivative?
- How can we use a tree diagram to guide us in applying the Chain Rule?

10.6: Gradient and Directional Derivatives

- The partial derivatives of a function $f(x, y)$ tell us the rate of change of $f(x, y)$ in the direction of the coordinate axes. How can we measure the rate of change of $f(x, y)$ in other directions?
- What is the gradient of a function and what does it tell us?

10.7: Optimization crit points occur when $\vec{\nabla} f(p) = \vec{0}$

- How can we find the points at which $f(x, y)$ has a local maximum, minimum, or saddle point?
- How can we determine whether critical points of $f(x, y)$ are local maxima or minima, or saddle points?

$D = f_{xx} f_{yy} - f_{xy}^2$

* $D_{\hat{u}} f(x,y) = \vec{\nabla} f(x,y) \cdot \hat{u}$ *

unit vector!!!

Exam 1 Outline (Important Concepts/Formulas)

- Slice of a function
- Level set / Contours
- Scalar vs Vector
- Vector addition, multiplication of a vector by a scalar
- Norm = Magnitude = Length of a vector
- Dot product
- Dot product (cosine version)
- Cross Product (determinant form)
- Cross Product (sine version)
- Parallel and Perpendicular Vectors
- Equation of a line in 3D given two points
- Equation of a plane given normal vector and a point
- Three-point method of finding planes (aka the cross product method)
- Limits of functions of two variables
- Finding a limit along a given curve
- Finding $f_x(a, b)$ and $f_y(a, b)$ using limit definition
- Finding partial derivatives algebraically
- Interpretations of first-order partials in terms of increasing/decreasing
- Computing second-order partials
- Clairaut's Theorem on the symmetry of mixed second-order partials.
- Interpretation of second-order partials
- Tangent plane and the Linearization of a function
- Differential of a function
- Compute new value of a function given old value and information about the differential
- Tree diagrams and the chain rule
- Use the chain rule to write down a derivative
- Directional derivatives: definition and interpretation
- Gradient: compute, plot, use
- Gradient and directional derivative and how they're related
- Critical points: definition, how to find
- Types of critical points, how to classify them
- Second derivative test, discriminant

Exit Ticket 1 Name: ______________________________ **Spring 2023**

You may use your notes on this exit ticket. **Be sure to show work and/or explain your reasoning.**

1. A car rental company charges a one-time application fee of 30 dollars, 50 dollars per day, and 0.11 dollars per mile for its cars.

   (a) Write a formula for the cost, C, of renting a car as a function $C = f(d, m)$ of the number of days d and the number of miles driven m.

   (b) Interpret the statement $f(4, 870) = \$365.70$ in the context of this problem, using at least one complete sentence.

Exit Ticket 2 Name: ______________________________ **Spring 2023**

You may use your notes on this exit ticket. **Be sure to show work and/or explain your reasoning.**

1. Let $\vec{\mathbf{u}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\vec{\mathbf{v}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 1\hat{\mathbf{k}}$. Find the following:

   (a) $\|\vec{\mathbf{u}}\|$

   (b) $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$

   (c) $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$

Exit Ticket 3 Name: ______________________________ **Spring 2023**

You may use your notes on this exit ticket. **Be sure to show work and/or explain your reasoning.**

1. Let $f(x, y) = 3x^2y - 2y^3x$.

   (a) Use the limit defintion of the partial derivative to compute $\frac{\partial f}{\partial x}(1, 2)$.

   (b) Compute $\frac{\partial f}{\partial y}$ algebraically (i.e. without using the limit defintion).

Exit Ticket 4 Name: ______________________________ **Spring 2023**

You may use your notes on this exit ticket. **Be sure to show work and/or explain your reasoning.**

1. Let $f(x, y) = 3x^2y - 2y^3x$.

   (a) Compute the gradient $\vec{\boldsymbol{\nabla}} f(x, y)$.

   (b) Compute the directional derivative $D_{\hat{\mathbf{u}}} f(1, 0)$.