Method of Lagrange Multipliers:

Target function: $f(x,y)$

Constraint: $g(x,y) = C$

$\Downarrow$

form a system of Equations

$$\begin{cases} g(x,y) = C \\ \vec{\nabla} f = \lambda \vec{\nabla} g \end{cases}$$

$\leftarrow$ multiple Eqns.

$\uparrow$ aux. variable / Lagrange multiplier

Ex

$f(x,y) = x^2 y$

$\underbrace{x^2+2y^2}_{g} = 6.$

$$\begin{cases} x^2+2y^2 = 6. \\ \vec{\nabla} f = \lambda \nabla g \ \circledast \end{cases}$$

$\uparrow$

$\nabla f = \langle 2xy, x^2 \rangle$

$\nabla g = \langle 2x, 4y \rangle$

$2xy = 2\lambda x$

$x^2 = 4\lambda y$

$$\begin{cases} x^2+2y^2=6. \\ 2xy=2\lambda x \\ x^2=4\lambda y \end{cases}$$

$\leftarrow$

$\boxed{x=\pm 2}$

$x^2 = 4\cdot(\pm 1)^2$

$\Rightarrow x^2=4$

$\hookrightarrow x=\pm 2.$

$\boxed{x^2=4\lambda^2}$

$4\lambda^2+2\lambda^2=6.$

$6\lambda^2=6$

$\lambda^2=1$

$\Downarrow$

$\lambda=\pm 1.$

$2xy=2\lambda x$

$2y=2\lambda \Rightarrow \boxed{y=\lambda}$

$\Downarrow \boxed{y=\pm 1,}$

mins $f(x,y)=-4$

maxes

$(+2,-1)$ $(+2,+1)$ $(-2,1)$, $(-2,-1)$

$f(x,y)=4$

Check: plug these points back into $f(x,y)=x^2y$