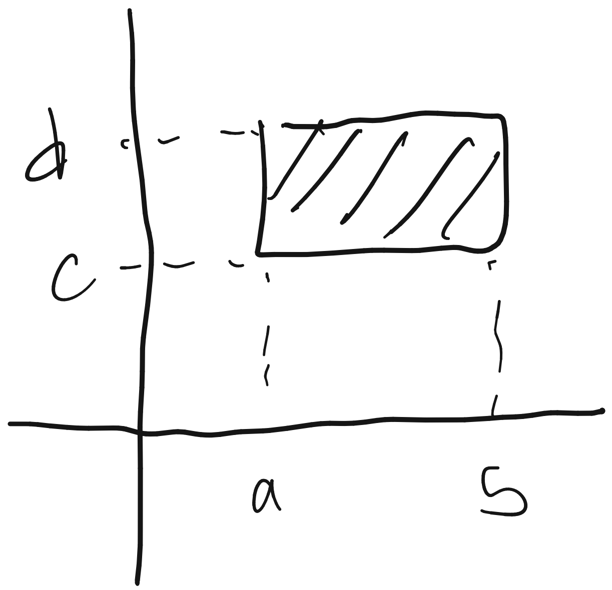
11.3 Double integrals over general regions:

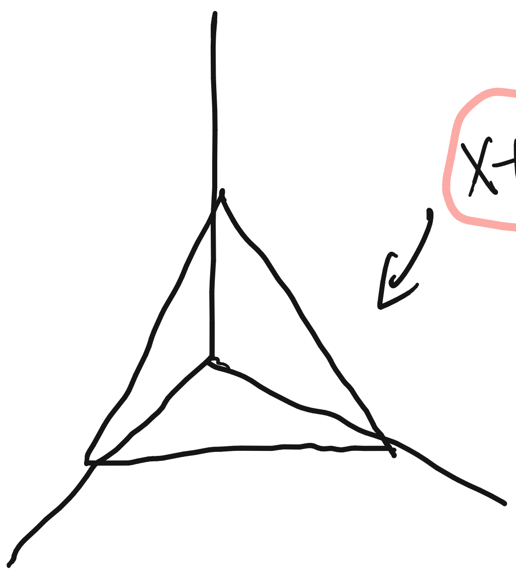


$$\rightsquigarrow \int_a^b \int_c^d f(x,y)\,dy\,dx$$

$$\parallel$$

$$\int_c^d \int_a^b f(x,y)\,dx\,dy$$

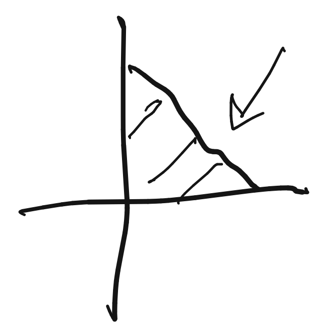
(Fubini's theorem)



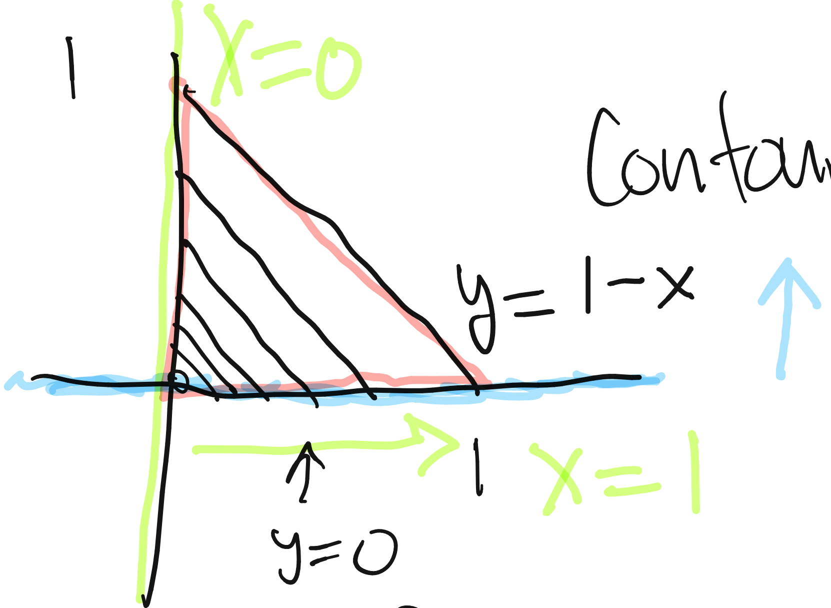
$x+y+z-1=0$ $\rightsquigarrow$ $z = 1-x-y$ ← height function.

Idea: top down view of this figure

looks like



integrate over this region.

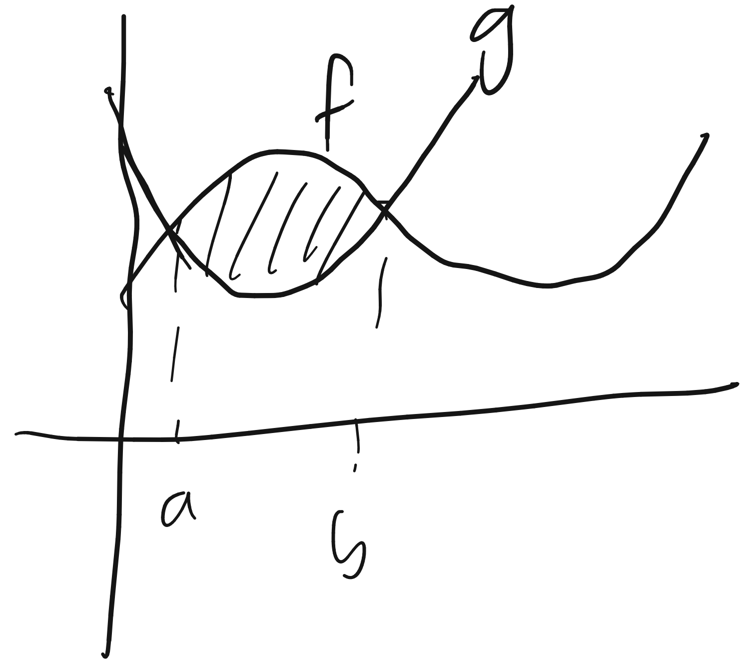


Contour picture of this Shape.

$$\text{Vol} = \iint_{\Delta} 1-x-y \; dA$$

the triangle!

$$\boxed{= \int_0^1 \int_0^{1-x} (1-x-y)\,dy\; dx}$$



$$A = \int_a^b f(x) - g(x)\; dx$$

$$\boxed{= \int_{x=a}^{x=b} \int_{y=g(x)}^{y=f(x)} 1 \; dy\, dx}$$

curves as bounds of integration!

Note: • Outer integral should contain only #s as bounds

• Inner integral can contain bounds that are functions of the "outer variable"

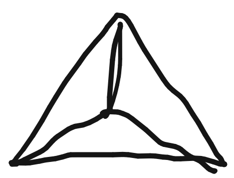
$$\int_0^1 \left[\int_0^{1-x} (1-x-y)\,dy\right] dx = \int_0^1 y - xy - \frac{y^2}{2}\Big|_{y=0}^{y=1-x} dx$$

$$= \int_0^1 \left[(1-x) - x(1-x) - \frac{1}{2}(1-x)^2\right] dx$$

$$= \int_0^1 \frac{1}{2} - x + \frac{1}{2}x^2 \quad dx$$

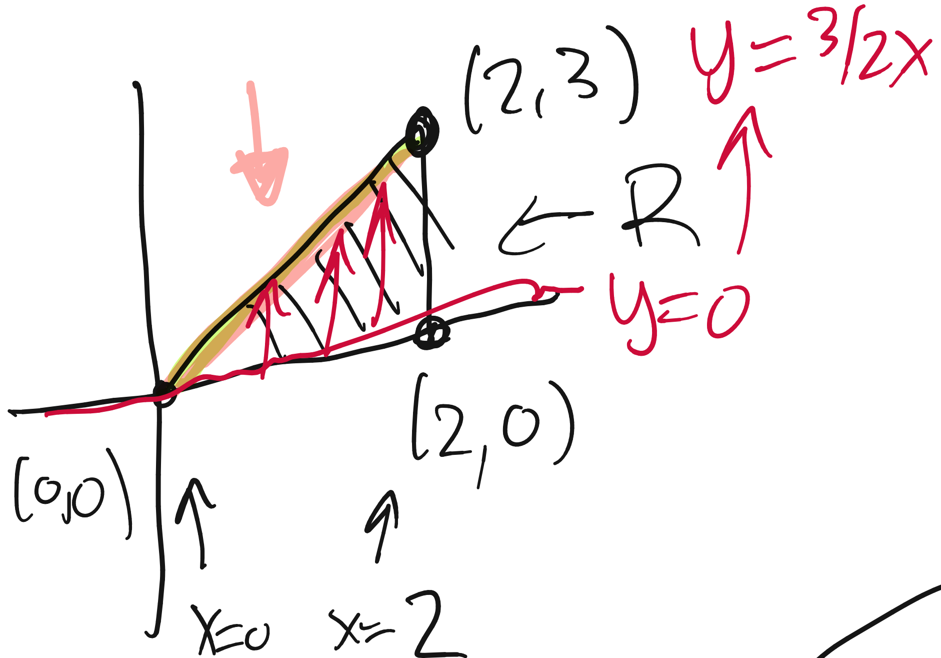
$$= \frac{1}{2}x - \frac{1}{2}x^2 + \frac{1}{6}x^3 \Big|_{x=0}^{x=1} = \frac{1}{2} - \frac{1}{2} + \frac{1}{6} - 0$$

$$= \boxed{1/6}$$

Vol () $= \frac{1}{6}$

Ex $f(x,y) = x^2y$

Region of integration is

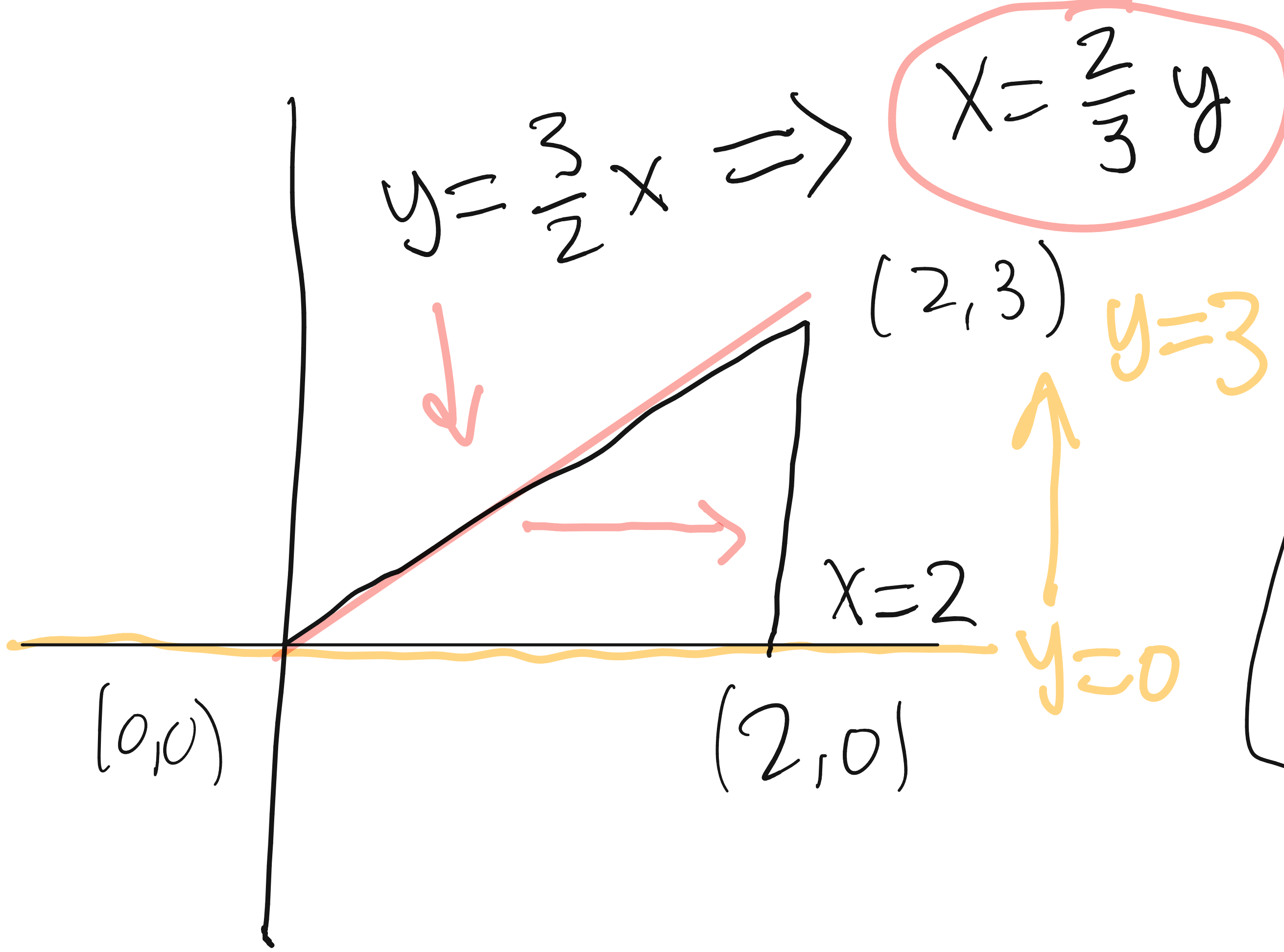


$$\frac{3}{2}(x-0)+0 = y$$

$$y = \frac{3}{2}x$$

$$\iint_R f(x,y)\,dA = \int_0^2 \int_0^{\frac{3}{2}x} x^2y\,dy\,dx$$

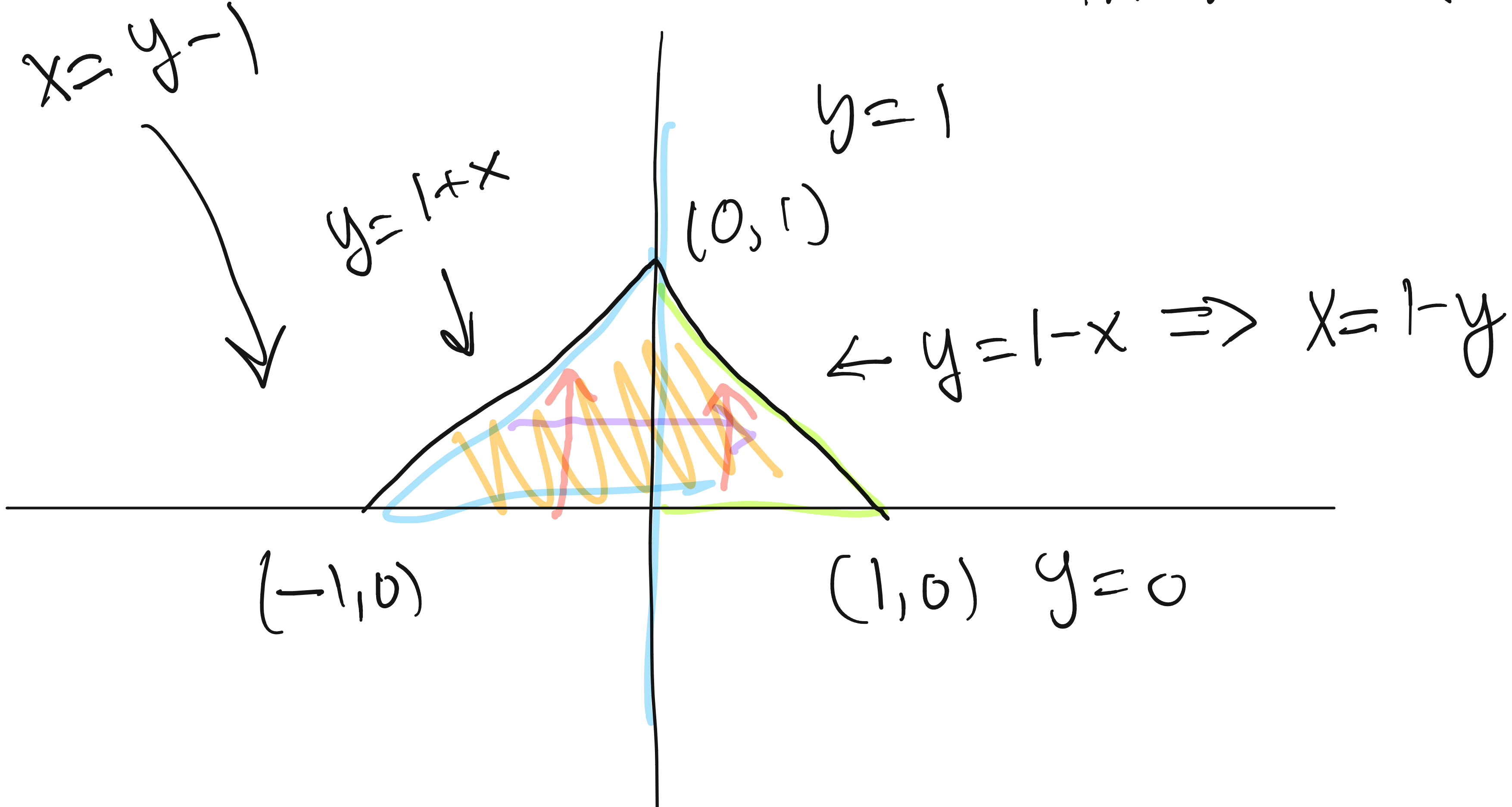
$$= 9/40$$



$$\int_0^3 \int_{\frac{2}{3}y}^{2} x^2 y \, dx \, dy$$

$= 9/40$

Integrate $f(x,y) = 1$ over this triangle.

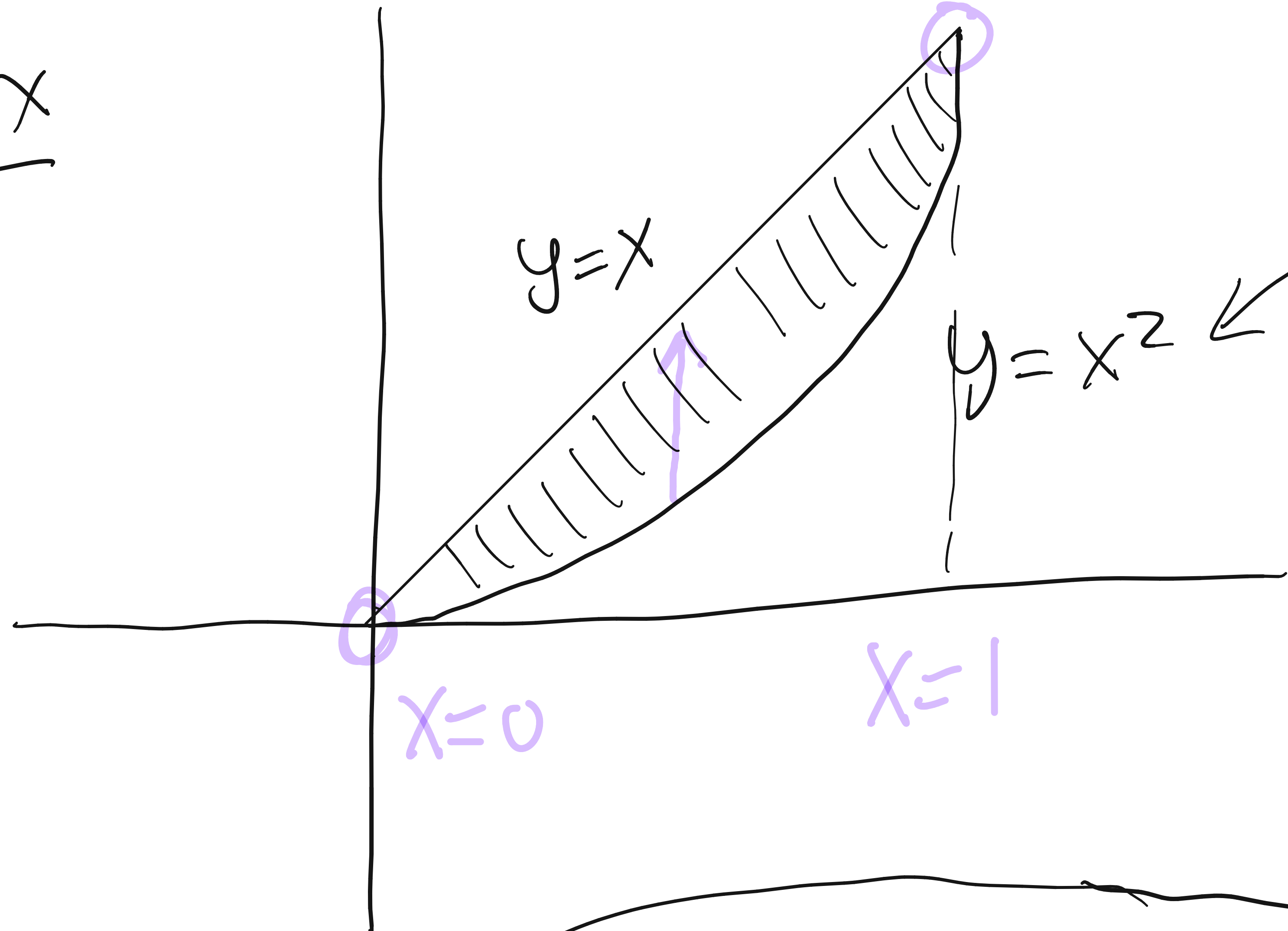


1 break up into two triangles (hard)

2 do x-integral first!

$$\int_0^1 \int_{y-1}^{1-y} 1 \, dx \, dy$$

Ex



$x = \sqrt{y}$

$z = 1 - x^2 - y^2.$

$x = x^2$

$\downarrow$

$x=0 \quad x=1$

$x^2 - x = 0 \Rightarrow x(x-1) = 0.$

Setup:

$$\int_0^1 \int_{x^2}^{x} 1 - x^2 - y^2 \, dy \, dx$$