# 9.6 Vector-Valued functions

<u>Notes:</u> · Exam 2 is next week. → Weds

Tentatively: 11.1 − 11.#, 9.6, 9.7

· WW 11.7 is due on Friday

a few weeks ago (> month ago) <u>linear functions:</u>

$$\vec{f}(t) = \vec{p}_0 + t\vec{v} = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

$\uparrow$ initial pos'n $\qquad$ $\uparrow$ direction of travel

$\vec{v} = \langle a, b, c \rangle$

This was our first Ex. of a <u>vector-valued function</u>

(V-V funcs.)

a <u>vector-valued func.</u> is a function

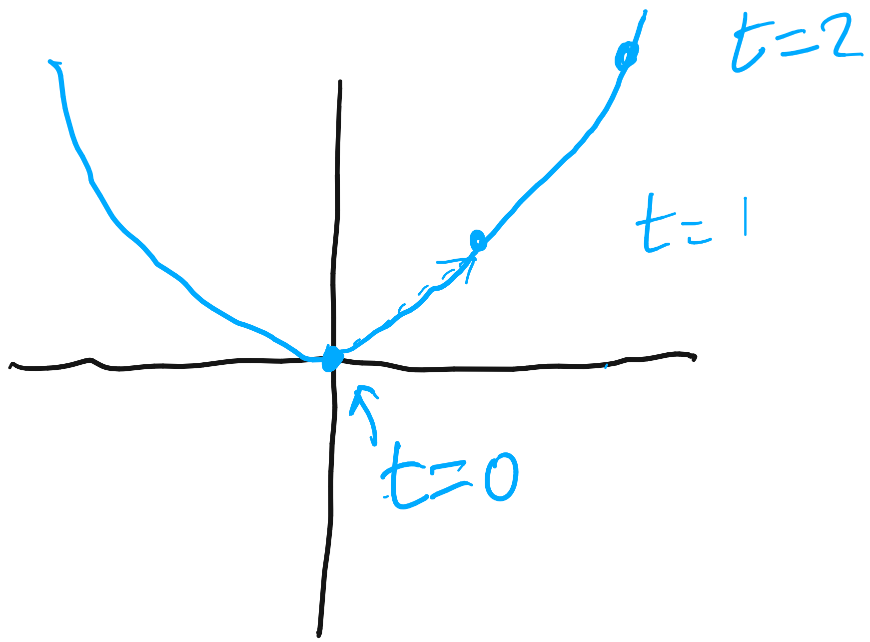
$\vec{f}(t)$ whose input is a real #

<u>Parameter</u> outputs are vectors.

these are also called <u>Parametric functions</u>

<u>graph</u> of a V-V function is a <u>Parametric curve</u>

Ex $\vec{f}(t) = \langle t, t^2 \rangle$




$t=0: \quad \vec{f}(0) = \langle 0, 0 \rangle$

$t=1 \quad f(1) = \langle 1, 1 \rangle$

$t=2 \quad f(2) = \langle 2, 4 \rangle$

$t=-1 \quad f(-1) = \langle -1, 1 \rangle$

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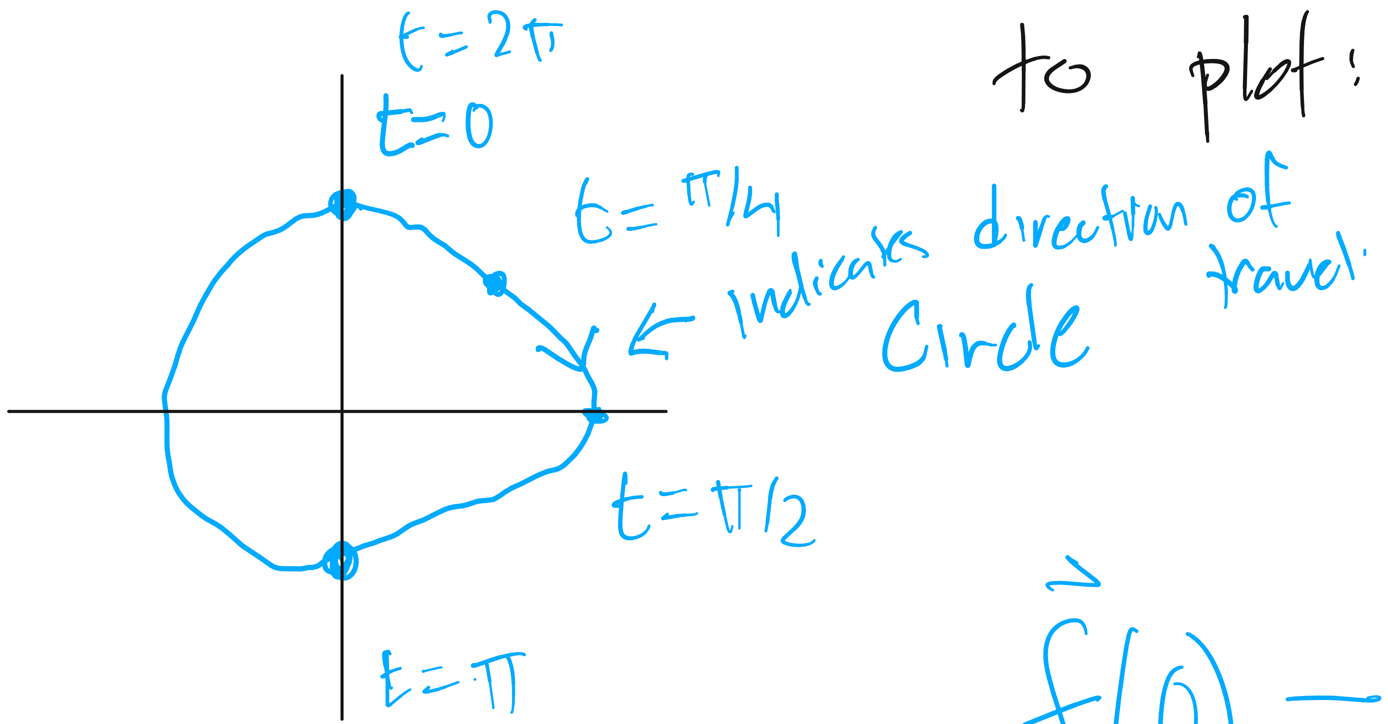
two ways:

Component functions

① unit-vector form: $\vec{f}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$

② list-form $\vec{f}(t) = \langle x(t), y(t), z(t) \rangle$

Ex $\vec{f}(t) = \langle \sin t, \cos t \rangle \qquad 0 \le t \le 2\pi$




to plot: Pick various t-values and then plug "connect-the-dots".

$$\vec{f}(0) = \langle \sin 0, \cos 0 \rangle$$
$$= \langle 0, 1 \rangle$$

$$\vec{f}(\pi/2) = \langle 1, 0 \rangle$$

$$f(\pi/4) = \langle \sin \pi/4, \cos \pi/4 \rangle$$
$$= \langle \tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2} \rangle$$

$$f(\pi) = \langle 0, -1 \rangle$$

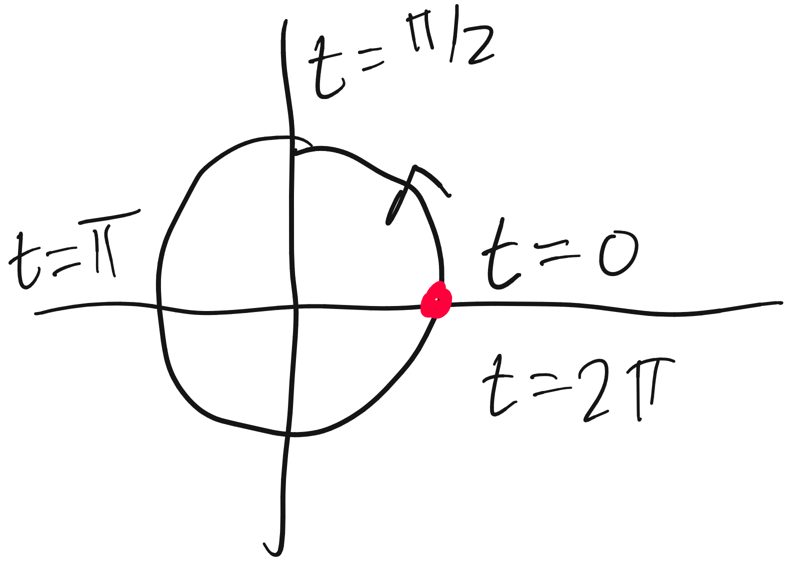
$$f(2\pi) = \langle 0, 1 \rangle.$$

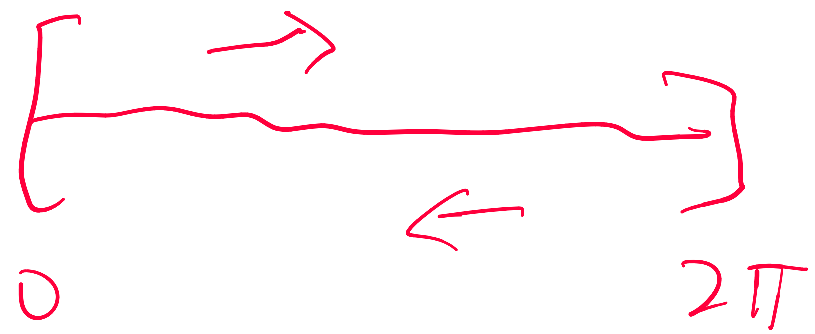
Standard Parameterization:

① Unit Circle:

$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

$0 \le t \le 2\pi$.




Want to go clockwise:

$$t \Rightarrow 2\pi - t.$$

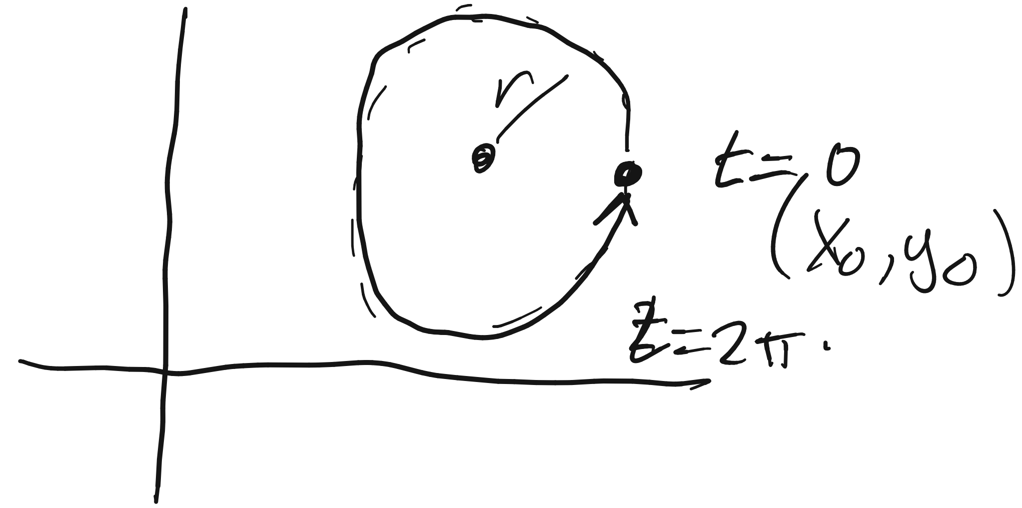
$$\vec{g}(t) = \langle \cos(2\pi - t), \sin(2\pi - t) \rangle$$

$$= \langle \cos t, -\sin t \rangle$$

2 "generic" circle

center point $(x_0, y_0)$

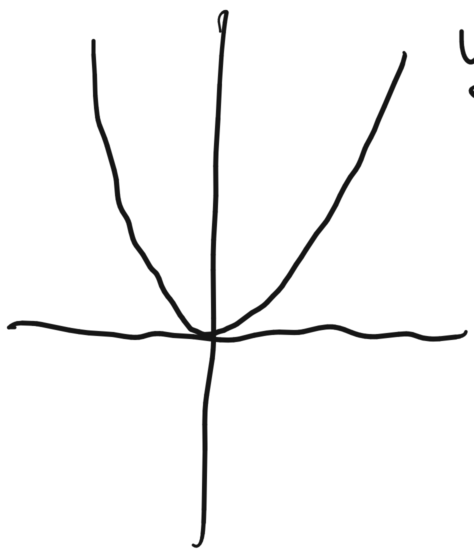
radius $r$




to param. the circle centered @ $(x_0, y_0)$ w/ radius $r$

$$\vec{f}(t) = \langle x_0 + r\cos t,\ y_0 + r\sin t \rangle$$

③ graph of a function $y = f(x)$.




$$(x, x^2)$$

$$\Downarrow \quad x = t$$

$$\langle t, t^2 \rangle =: \vec{f}(t)$$

In gen'l given $y = f(x)$ can build a param. func

$$\vec{g}(t) = \langle \underset{\underset{x}{\uparrow}}{t}, \underset{\underset{y}{\uparrow}}{f(t)} \rangle$$

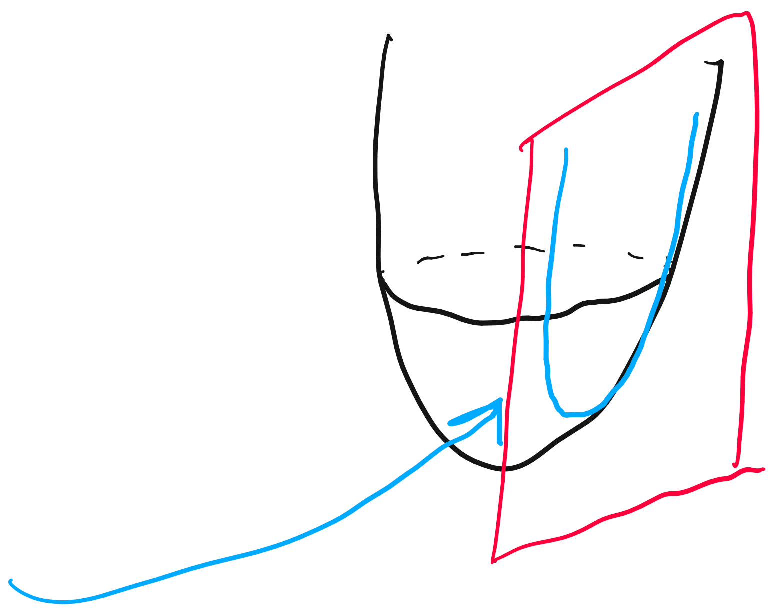
④ traces of functions:

if $z = f(x,y)$ let's do the $y=1$ trace.

$$z = f(x, 1)$$

$$\langle t, 1, f(t,1) \rangle$$

x y z.



to Parameterize a shape is to find

a Parametric function whose graph is that shape.