Exam 2 review

Exam 2 is tomorrow (in-class)

- Lagrange multipliers will be on the exam.

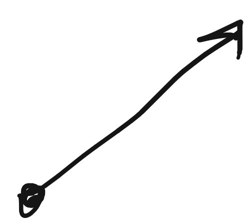
- Double, triple ints will be on exam.

$$f = f(x,y,z)$$

$$\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$\vec{\nabla} f(\vec{p}) = \left\langle \frac{\partial f}{\partial x}(\vec{p}), \frac{\partial f}{\partial y}(\vec{p}), \frac{\partial f}{\partial z}(p) \right\rangle.$$

Think: $\vec{\nabla} f(\vec{p})$ is a vector based @ $\vec{p}$



P

- Perpendicular to contours
- points in dir. of greatest ascent.

In Calc 1

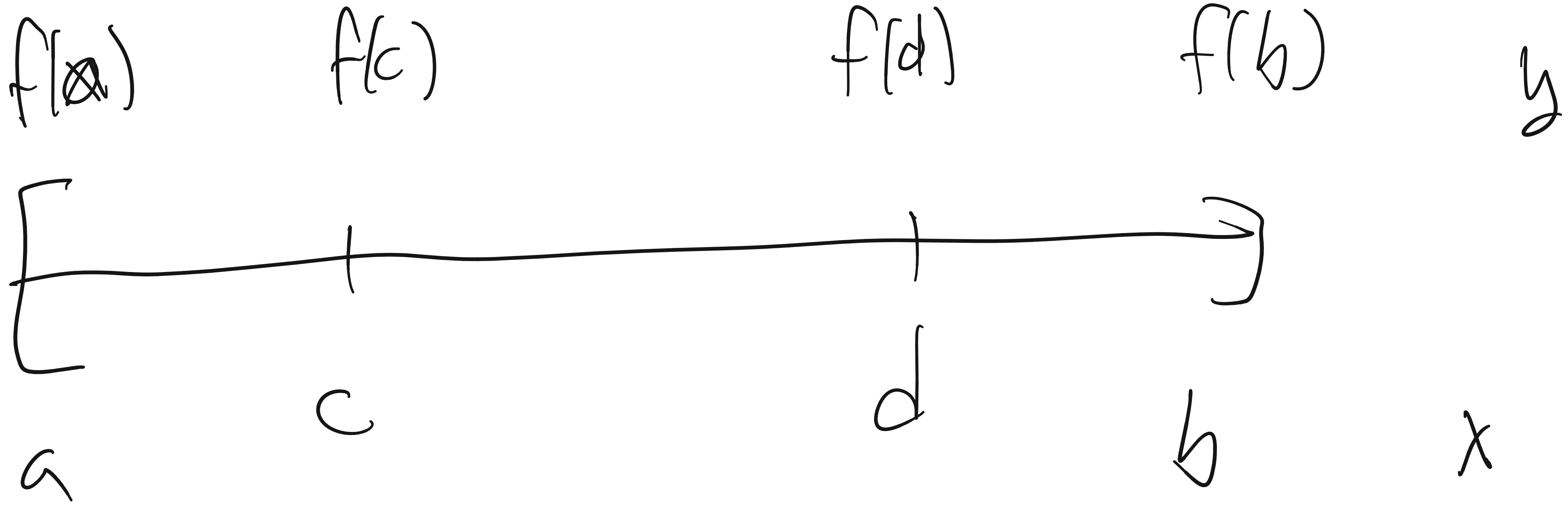
finding extrema for $y=f(x)$ on $[a, b]$

① find critical values for $y=f(x)$ by first deriv test

throw away Extrema not w/in $[a, b]$.

② 2^{nd} deriv. test tells you about local behavior (ie what happens near criti values).

③ function tells you what happens @ endpoints.



Cat 3 POU:

(Lagrange mult problems):

$\vec{\nabla} f(\vec{p}) = \lambda \vec{\nabla} g(\vec{p})$ so solutions look like

$(\vec{p}, \lambda)$

test all soln's to see if they're mins/maxes by

Evaluate your func. @ that point.

$$\underbrace{f(\vec{p}_1)}_{w_1}, \quad \underbrace{f(\vec{p}_2)}_{w_2}, \quad \underbrace{f(\vec{p}_3)}_{w_3}$$

if $w_2 > w_1, w_3$ $\quad f(p_2)$ is max value.

Initials: $\boxed{}$ (Practice Exam 2A) Math 208, Exam 2

Problem 3 (20 points). Use the method of Lagrange multipliers to find the maximum value taken by the function $f(x, y, z) = \underline{3yz - x^2}$ on the plane $\underbrace{-2x + 6y + 6z = 11}_{g}$.

① ∇f, ∇g.

(1) $\vec{\nabla} f = \langle -2x, 3z, 3y \rangle$

$\vec{\nabla} g = \langle -2, 6, 6 \rangle$

② Setup Sys. of Eqns:
$$\begin{cases} -2x + 6y + 6z = 11 \\ \underline{\vec{\nabla} f = \lambda \vec{\nabla} g} \end{cases}$$

$-2x = \lambda(-2)$, $3z = 6\lambda$, $3y = 6\lambda$

$$\begin{cases} -2x+6y+6z=11 \\ -2x=-2\lambda \\ 3z=6\lambda \\ 3y=6\lambda \end{cases}$$

$-2x=-2\lambda \rightsquigarrow x=\lambda$

$\left.\begin{matrix} 3z=6\lambda \\ 3y=6\lambda \end{matrix}\right\} \Rightarrow z=2\lambda,\ y=2\lambda$

$x=1/2,$

$y=1,\ z=1.$

$(*)$

$$-2\lambda+6(2\lambda)+6(2\lambda)=11$$

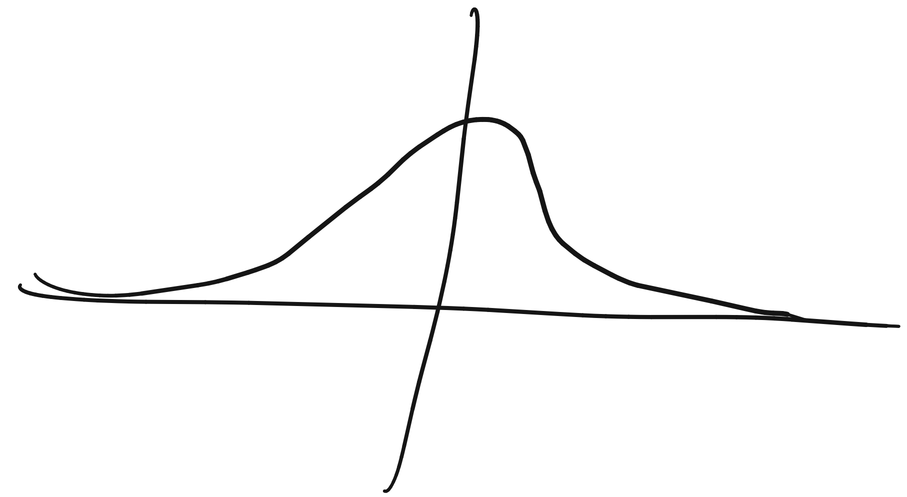
$$-2\lambda+12\lambda+12\lambda=11$$

$$22\lambda=11 \Rightarrow \lambda=11/22=1/2$$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx =: I$$

$$\| $$

$$\sqrt{\pi}$$

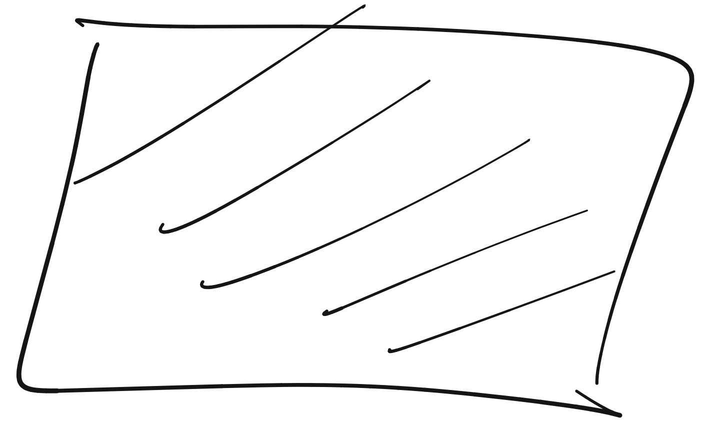


e^{-x^2} is related to Normal distb'n.

Trick rather than computing I, we'll compute I^2 instead.

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2 = \overbrace{\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)}^{I}\overbrace{\left(\int_{-\infty}^{\infty} e^{-y^2}\,dy\right)}^{I}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dy\,dx$$

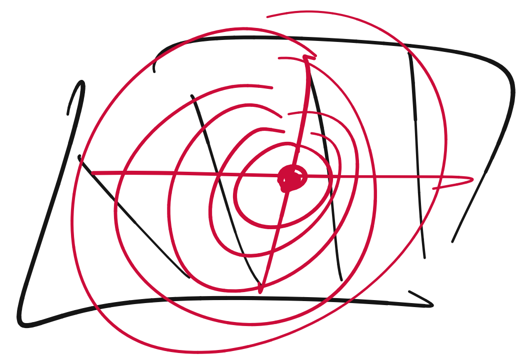


$\mathbb{R}^2$

$$= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}} \underset{\uparrow\\ f(x,y)}{e^{-(x^2+y^2)}} \underbrace{dy\,dx}_{dA}$$

Change to polar coords!

$$= \int_{0}^{2\pi}\int_{0}^{\infty} e^{-r^2}\; r\,dr\,d\theta$$



$$= \int_0^{2\pi} \int_0^{\infty} r e^{-r^2} \, dr \, d\theta$$

$$u = -r^2$$

$$du = -2r \, dr.$$

$$= \int_0^{2\pi} \int_0^{-\infty} -\frac{1}{2} e^u \, du \, d\theta.$$

"flippy dippy trick"

$$= \int_0^{2\pi} \int_{-\infty}^{0} \frac{1}{2} e^u \, du \, d\theta$$

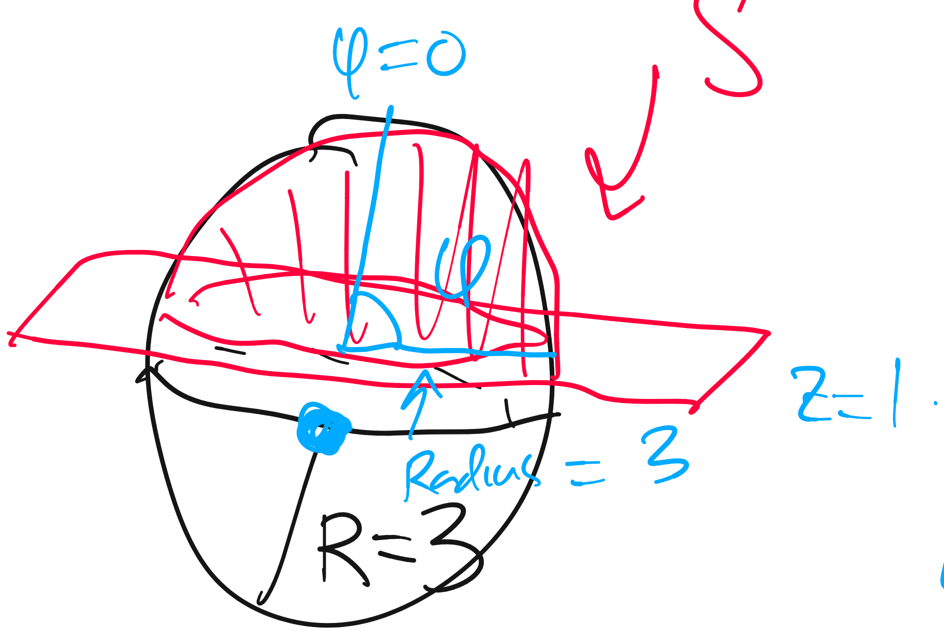
$$= \int_0^{2\pi} \cdot \frac{1}{2} \left(e^0 - \cancel{e^{-\infty}} \to 0 \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \, d\theta = \frac{2\pi}{2} - \frac{0}{2} = \boxed{\pi}$$

$$I^2 = \pi \implies I = \sqrt{\pi}$$

Ex Setup an integral that computes the volume of the solid bounded below by the plane $z=1$, above by the ball

$$0 \leq x^2+y^2+z^2 \leq 9$$



$$\boxed{z=\rho\cos\varphi}$$

$$\text{Vol}(S) = \iiint_S dV$$

$\cancel{\rho \neq 0}$. $\rho = ? \longrightarrow \rho = 3$.

$1 = 3\cos\varphi \Rightarrow \varphi = \arccos(1/3)$

let's do it in sph. coords.

$$\int_0^{2\pi} \int_0^{\arccos\left(\frac{1}{3}\right)} \int_{\sec\varphi}^{3} 1 \, \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

θ $\quad \varphi$

$$z = \rho\cos\varphi \Rightarrow 1 = \rho\cos\varphi \Rightarrow \rho = \sec\varphi$$

$$= 1/\cos\varphi$$

$$\begin{cases} g(\vec{x}) = k \\ \vec{\nabla f} = \lambda \vec{\nabla g} \end{cases}$$ Solve for (x, y, z, λ)

Exam 2 Outline (Motivating Questions)

10.8 Lagrange Multipliers

- What geometric condition enables us to optimize a function $f = f(x, y)$ subject to a constraint given by $k = g(x, y)$, where k is a constant?
- How can we exploit this geometric condition to find the extreme values of a function subject to a constraint?

$\iint_R f\,dA$ 11.1 Double Integrals

- ~~What is a double Riemann sum?~~ No Riemann Sums on this Exam.
- How is the double integral of a continuous function $f = f(x, y)$ defined?
- What are two things the double integral of a function can tell us?

$\int_a^b \int_c^d f\,dx\,dy$ 11.2 Iterated Integrals Fubini's Theorem

- How do we evaluate a double integral over a rectangle as an iterated integral, and why does this process work?

11.3 Double integrals over general regions

- How do we define a double integral over a non-rectangular region?
- What general form does an iterated integral over a non-rectangular region have?

$\int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$

11.4 Applications of double integrals

- If we have a mass density function for a lamina (thin plate), how does a double integral determine the mass of the lamina?
- How may a double integral be used to find the area between two curves?
- Given a mass density function on a lamina, how can we find the lamina's center of mass?

11.5 Double integrals in polar coordinates "Polar trick" for integrals $\int e^{x^2}\,dx$

- What are the polar coordinates of a point in two-space?
- How do we convert between polar coordinates and rectangular coordinates?
- What is the area element in polar coordinates?
- How do we convert a double integral in rectangular coordinates to a double integral in polar coordinates?

11.7 Triple integrals

- How are a triple Riemann sum and the corresponding triple integral of a continuous function $f = f(x, y, z)$ defined?
- What are two things the triple integral of a function can tell us?

11.8 Triple integrals in Cylindrical and Spherical coordinates

- What are the cylindrical coordinates of a point, and how are they related to Cartesian coordinates?

- What is the volume element in cylindrical coordinates? How does this inform us about evaluating a triple integral as an iterated integral in cylindrical coordinates?
- What are the spherical coordinates of a point, and how are they related to Cartesian coordinates?
- What is the volume element in spherical coordinates? How does this inform us about evaluating a triple integral as an iterated integral in spherical coordinates?

9.6: Vector-Valued Functions

- What is a vector-valued function? What do we mean by the graph of a vector-valued function?
- What is a parameterization of a curve in $\mathbb{R}^2$? In $\mathbb{R}^3$?
- What can the parameterization of a curve tell us?

9.7: Derivatives and Integrals of Vector-Valued Functions

- What do we mean by the derivative of a vector-valued function and how do we calculate it? *
- What does the derivative of a vector-valued function measure?
- What do we mean by the integral of a vector-valued function and how do we compute it? *

Standard Examples : lines,

Circles,

Curve $y = f(x)$

Exam 2 Outline (Important Concepts and Formulas)

- Method of Lagrange Multipliers
- Interpretation of λ in a Lagrange multipliers question
- Determining if a solution to Lagrange multipliers question is min or max
- Double Integrals (numerically)
- Double integrals over rectangles
- Double integrals over general regions
- Computing double integrals
- Polar coordinates
- dA in polar coordinates
- Polar to Cartesian and vice-versa
- Mass, area, and center of mass computations in 2-D
- Triple integrals over cuboids
- Triple integrals over general regions
- Computing triple integrals
- Cylindrical Coordinates
- dV in cylindrical coordinates
- Cartesian to Cylindrical coordinate conversions (and vice-versa)
- Spherical Coordinates
- dV in spherical coordinates
- Cartesian to Spherical coordinates conversions (and vice-versa)
- Vector-valued functions
- Plots of vector-valued functions
- Forms of vector-valued functions
- Derivatives of vector-valued functions
- Interpretations of derivatives/integrals of vector-valued functions
- Integrals of vector-valued functions
- Standard parameterizations
    - Lines
    - (general) circles
    - Unit Circle
    - Graphs of functions of the form $y = f(x)$
- Derivatives of vector-valued functions
- Interpretations of first, second derivatives of a vector-valued function
- Integrals of vector-valued functions