AC 12.9 / OS 6.6 part 2

Last time...

Surface / Flux integral

$$\text{flux} = \underbrace{\iint_S \vec{F} \cdot \vec{n}\, dA}_{} = \iint_S \vec{F} \cdot d\vec{A}$$

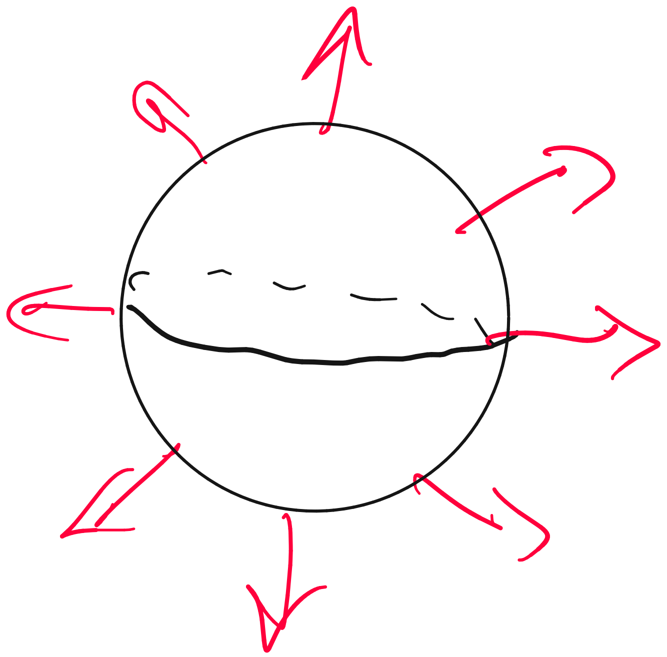
F: Smooth v-field

$\vec{n} = \vec{r}_s \times \vec{r}_t$ "normal vectors"

S surf. parameterized by $\vec{r}(s,t)$

A surface S is <u>oriented</u> if we can draw our choice of normal vectors all point in (roughly speaking) the same direction.

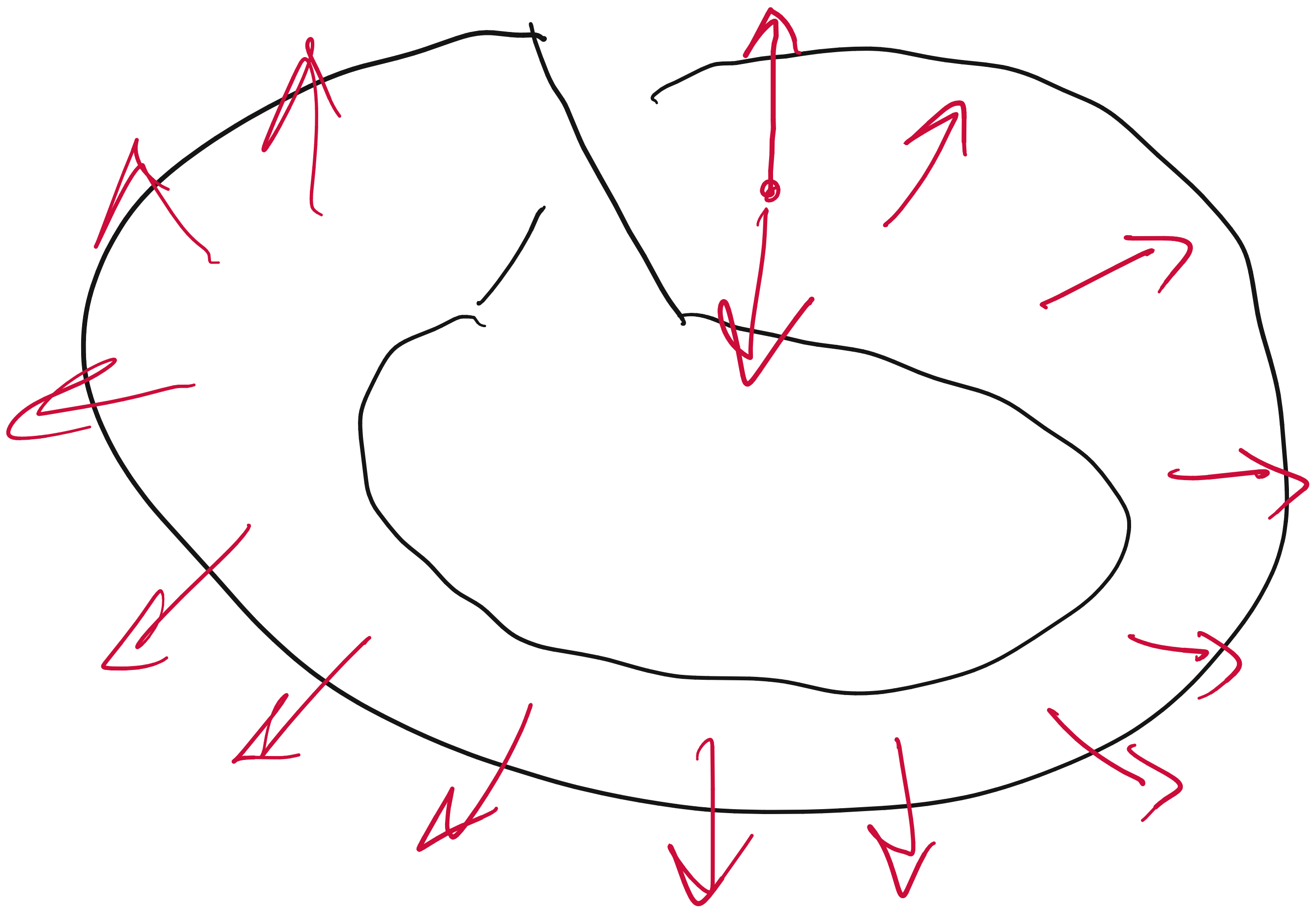
Ex



outwards orientation

$\vec{n}$

upwards orientation.



Not orientable.

bad :(

All surfaces from here on will be orientable

ie can find a choice of normal vectors that

point "Consistently".

Ex (WW 12.9-1 #2)

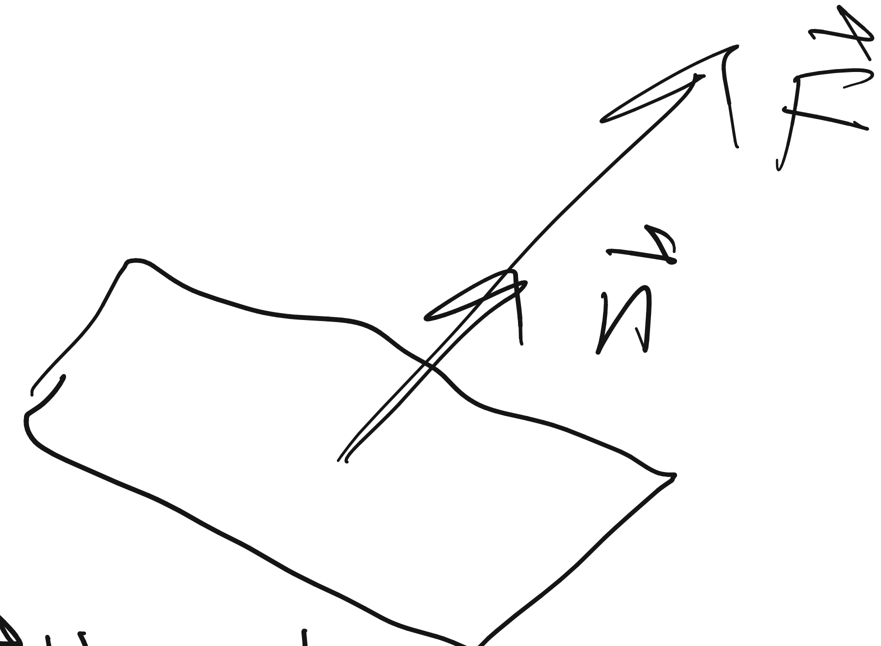
$\langle 1,1,1 \rangle$

$\downarrow$

$F = \langle 3,3,3 \rangle$

$S =$ disk of radius 3 in plane $x+y+z=1$.

First: note that $F \perp S$



Why? $F \parallel \vec{n}$

Pick a param. of S st $\|\vec{n}\| = 1$.

$$\vec{F} \cdot \vec{n} = \|F\| \, \cancel{\|n\|}^{1} \, \cancel{\cos(\theta)}^{1}$$

So: flux $= \iint_S \vec{F}\cdot\vec{n}\,dA = \iint_S \|F\|\,dA$ (here)

$F = \langle 3, 3, 3 \rangle$

$\|F\| = \sqrt{3^2+3^2+3^2} = \sqrt{27}$

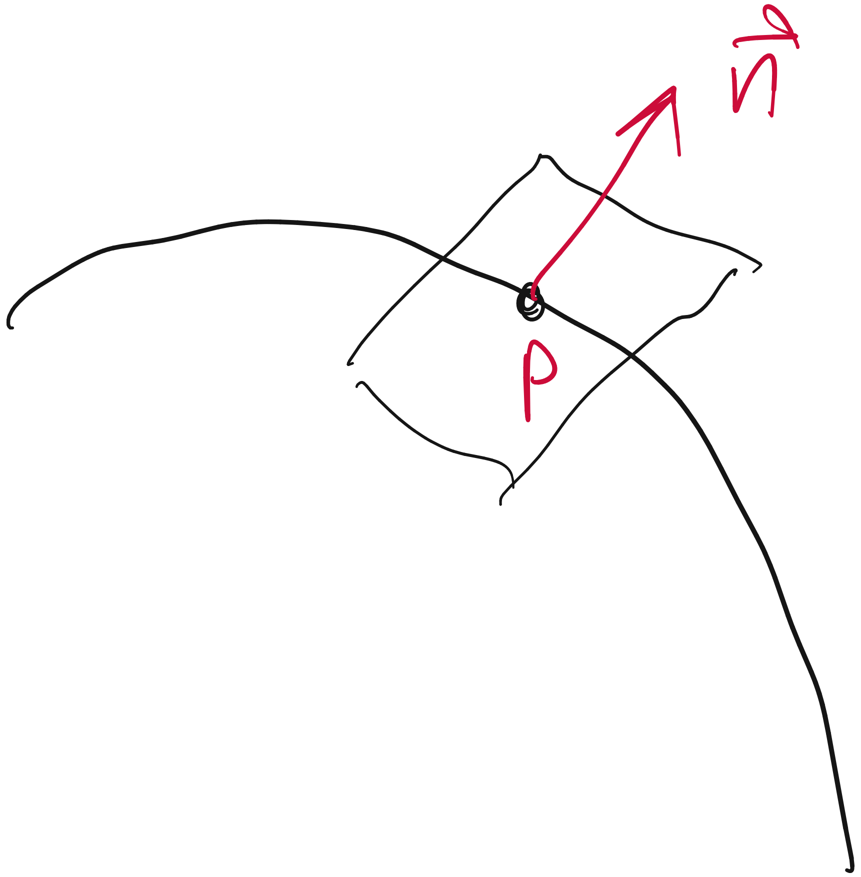
$\Rightarrow$ flux $= \iint_S \sqrt{27}\,dA = \sqrt{27}\iint_S dA = \sqrt{27}\cdot$ Area

$\boxed{= \sqrt{27}\cdot 9\pi}$

<u>Fact:</u> If S is an oriented surface w/ normal vectors $\vec{n}$ and $\vec{F}$ is a v.field w/ $\vec{F} \parallel \vec{n}$ (Equivalently, $S \perp \vec{F}$) then:

$$\text{flux} = \iint_S \vec{F} \cdot \vec{n} \, dA = \iint_S \|\vec{F}\| \, dA$$

<u>NB:</u> In the prev. Ex, $\|F\|$ was constant. this is <u>not</u> always the case.



$F \perp S$ @ p if & only if

$F \parallel \vec{n}$

$\vec{v} \parallel \vec{w}$ if & only if

$\vec{v} = \lambda \vec{w}$ some $\lambda \neq 0$.

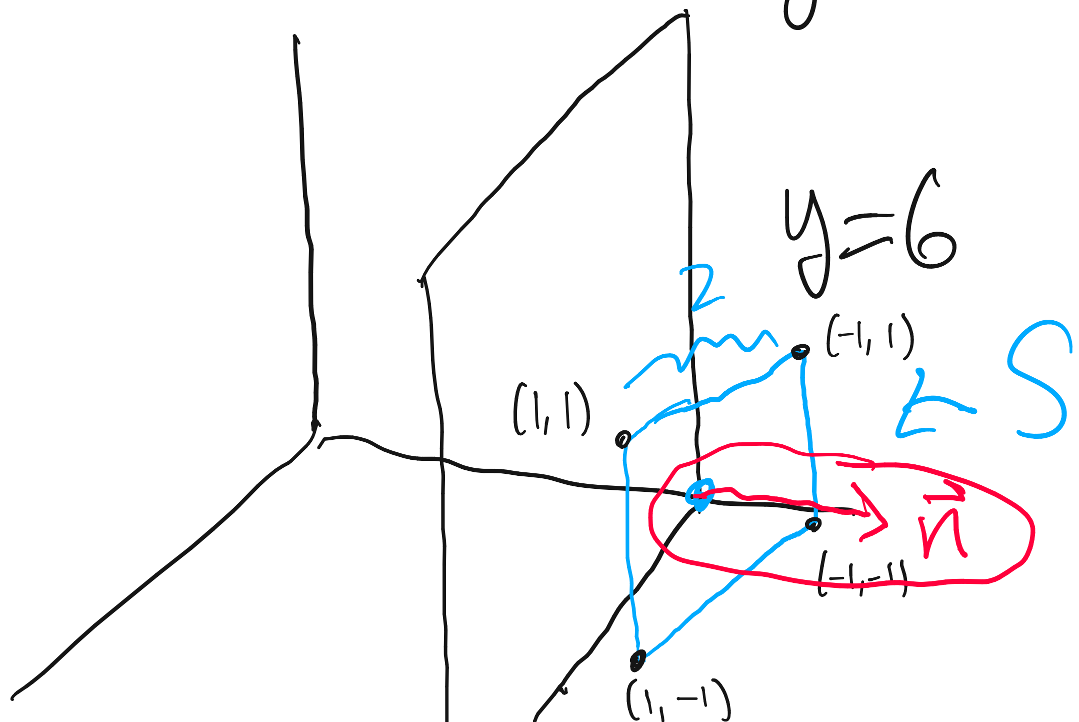
Ex (WW 12.9-1 #3)

$F = \langle 4, 3x^2, -3 \rangle$

S is square of side length 2 on plane $y=6$

centered on y-axis w/ sides parallel to x, z axes.

Oriented in +y-direction.



$r(s,t) = \boxed{\langle t, 6, s \rangle}$

$-1 \leq t \leq 1$

$-1 \leq s \leq 1$

$$\text{Flux} = \iint_S \vec{F} \cdot \vec{n}\, dA$$

$$n = r_s \times r_t$$

$$r_s = \langle 0, 0, 1 \rangle = \hat{k}$$

$$r_t = \langle 1, 0, 0 \rangle = \hat{\imath}$$

$x(s,t) = t$

$y(s,t) = 6$

$z(s,t) = s$

$$r_s \times r_t = \hat{k} \times \hat{\imath} = \hat{\jmath} = \langle 0, 1, 0 \rangle$$

match.

$$\vec{F}(\vec{r}(s,t)) = \langle 4, 3t^2, -3 \rangle$$

$$\vec{F}(\vec{r}(s,t)) \cdot \vec{n} = 3t^2$$

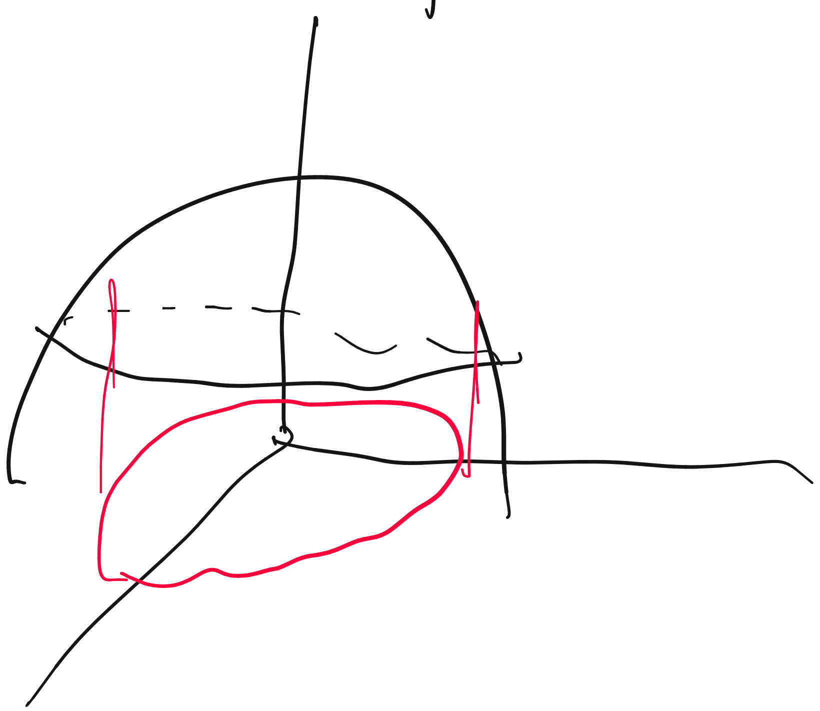
$$\text{Flux} = \iint_S \vec{F} \cdot \vec{n}\, dA = \int_{1}^{1} \int_{-1}^{1} 3t^2 \, dt\, ds = 4$$

Ex WW 12.9-2 #3

$\vec{F} = \langle x, y, 0 \rangle$

S is part of surf. $z = 9 - x^2 - y^2$ lies above disk of radius 3 centered @ origin.

Oriented upwards.



$$\begin{array}{cccc} & x & y & f(x,y) \\ \vec{r}(s,t) = & \langle s\cos t, & s\cdot\sin t, & 9-s^2 \rangle \end{array}$$

$0 \le s \le 3, \quad 0 \le t \le 2\pi.$

s plays role of r

t plays role of θ.

$\vec{r}_s = \langle \cos t,\ \sin t,\ -2s \rangle$

$\vec{r}_t = \langle -s\sin t,\ s\cos t,\ 0 \rangle$

orientation matches b/c $s \geq 0$

$\vec{r}_s \times \vec{r}_t = \langle 2s^2\cos t,\ 2s^2\sin t,\ s \rangle = \vec{n}$

$F = \langle x, y, 0 \rangle$

$F(r(s,t)) = \langle s\cdot\cos t,\ s\cdot\sin t,\ 0 \rangle$

$\vec{F}(\vec{r}(s,t)) \cdot \vec{n} = 2s^3\cos^2 t + 2s^3\sin^2 t + 0 = 2s^3$

$2s^3(\cos^2 t + \sin^2 t)$

$$\text{flux} = \iint_S \vec{F}\cdot\vec{n}\, dA = \int_0^{2\pi}\int_0^3 2s^3\, ds\, dt.$$

$$= \boxed{81\pi}$$