Exam 3 is tomorrow!

$\boxed{\text{Big topics:}}$

Arc length	Curl
Vector fields	divergence
Line integrals	Green's theorem
Parametric Surfaces	Path independence
flux integrals	FTC line integrals

$\boxed{\text{"Standard trick"}}$ for parametric curves:

$$y = f(x) \Rightarrow \vec{r}(t) = \langle t, f(t) \rangle$$

$a \leq x \leq b$

t bounds = x bounds

$a \leq t \leq b$

$\underline{Ex}$ $f(x) = x^2 + 3x + 4$ $\quad -3 \leq x \leq 17$

$$\Updownarrow$$

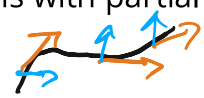
$\vec{r}(t) = \langle t, t^2 + 3t + 4 \rangle$; $-3 \leq t \leq 17$.

$$s=\int_a^b \|\vec{r}\,'(t)\|\,dt = \int_a^b \sqrt{(x')^2+(y')^2+(z')^2}\,dt$$

Exam 3 Outline (Motivating Questions)

– 9.8: Arc Length
- How can a definite integral be used to measure the length of a curve in 2- or 3-space?
- Why is arc length useful as a parameter?

→ 12.1: Vector Fields
- What is a vector field? gradient vector fields, P-I v. fields
- What are some familiar contexts in which vector fields arise?
- How do we draw a vector field?
- How do gradients of functions with partial derivatives connect to vector fields?



12.2: The Idea of a Line integral
- What is an oriented curve and how can we represent one algebraically?
- What is the meaning of the line integral of a vector-valued function along a curve and how can we estimate if its value is positive, negative, or zero?
- What are important properties of the line integral of a vector-valued functions along a curve?

 closed curve 

12.3: Using Parameterizations to Compute Line Integrals

$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

- How can we use a parametrization of an oriented curve C to calculate $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$
- How does the parametrization chosen for an oriented curve C alter the value of the line integral $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$
- What can be said about the line integral of a vector field along two different oriented curves when the curves have the same starting point and same ending point?

12.5: Path Independence and FTC for Line Integrals gradient v.fields = P.I v.fs.
- What characteristic of a vector field $\boldsymbol{F}$ will make $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ have the same value for every oriented curve from a point P to a point Q?
- What special properties do gradient vector fields have?
- Given a gradient vector field $\boldsymbol{F}$, how can we efficiently find a potential function f so that $\boldsymbol{F} = \nabla f$

12.7: The Curl of a Vector Field
- What is meant by rotation of a vector field in a plane?
- How can a two-dimensional measurement of rotation be generalized to work in three dimensions?
- How can the rotational strength of a vector field be measured?

$\boxed{\mathbb{R}^2 \text{ only}}$ 12.8: Green's Theorem
- How can we calculate the circulation of a two-dimensional vector field $\boldsymbol{F}$ around a closed curve when $\boldsymbol{F}$ is not path-independent?

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_R \text{curl}(F)\,dA$$

C has to be simple closed curve

- What is the meaning of the double integral of the circulation density of a smooth two-dimensional vector field on a region R bounded by a closed curve that does not intersect itself?

11.6: Surfaces Defined Parametrically and Surface Area

- What is a parameterization of a surface?
- How do we find the surface area of a parametrically defined surface?

12.9: Flux Integrals

- How can we measure how much of a vector field flows through a surface in space?
- How can we calculate the amount of a vector field that flows through common surfaces, such as the graph of a function $z = f(x, y)$

12.6: The Divergence of a Vector Field

- How can you measure where a vector field is created (or destroyed)?
- How can you measure where a vector field's strength is increasing or decreasing?
- What does the divergence of a vector field measure and how can you visually estimate whether the divergence of a vector field is positive or negative?

$$F = \langle P, Q, R \rangle$$

$$\text{div}(F) = P_x + Q_y + R_z$$

Exam 3 Outline (Important Concepts and Formulas)

- Arc-length formula
- Reparametrization and arc-length parameterization
- Vector fields
- How to plot a vector field
- Gradient vector fields
- Line integrals with and without using parameterizations
- Work done by a vector field
- Path-independence
- How to tell if a vector field is path-independent
- FTC for Line Integrals
- Potential functions and how to find them
- How to compute curl of a vector field in 2d, 3d
- Interpretations of curl
- What does it mean if $curl(\boldsymbol{F}) = \mathbf{0}$? What if curl is non-zero?
- What is a closed curve?
- What is a simply connected region?
- What is a simple closed curve?
- What is Green's theorem and when can we use it?
- Circulation & circulation density of a vector field
- Parametrizations of surfaces
- Common examples:
    - Graphs of functions of the form $z = f(x, y)$
    - Surfaces of revolution
    - Cylinders
    - Spheres
    - Planes
    - Cones
- Surface area formula from a parameterization
- What is the normal vector of a surface? How is it computed and how do you visualize it?
- What does it mean for a surface to be oriented?
- Flux (i.e. Surface) integrals
- How to compute surface integrals with(out) using a parameterization
- Divergence of a vector field
- Source-free / Divergence-free
- Interpretations of divergence
- Approximation of flux by divergence and small surfaces

Parametric Surfaces:

$$z = f(x,y) \qquad (\text{or} \quad y = f(x,z) \quad \text{etc.})$$

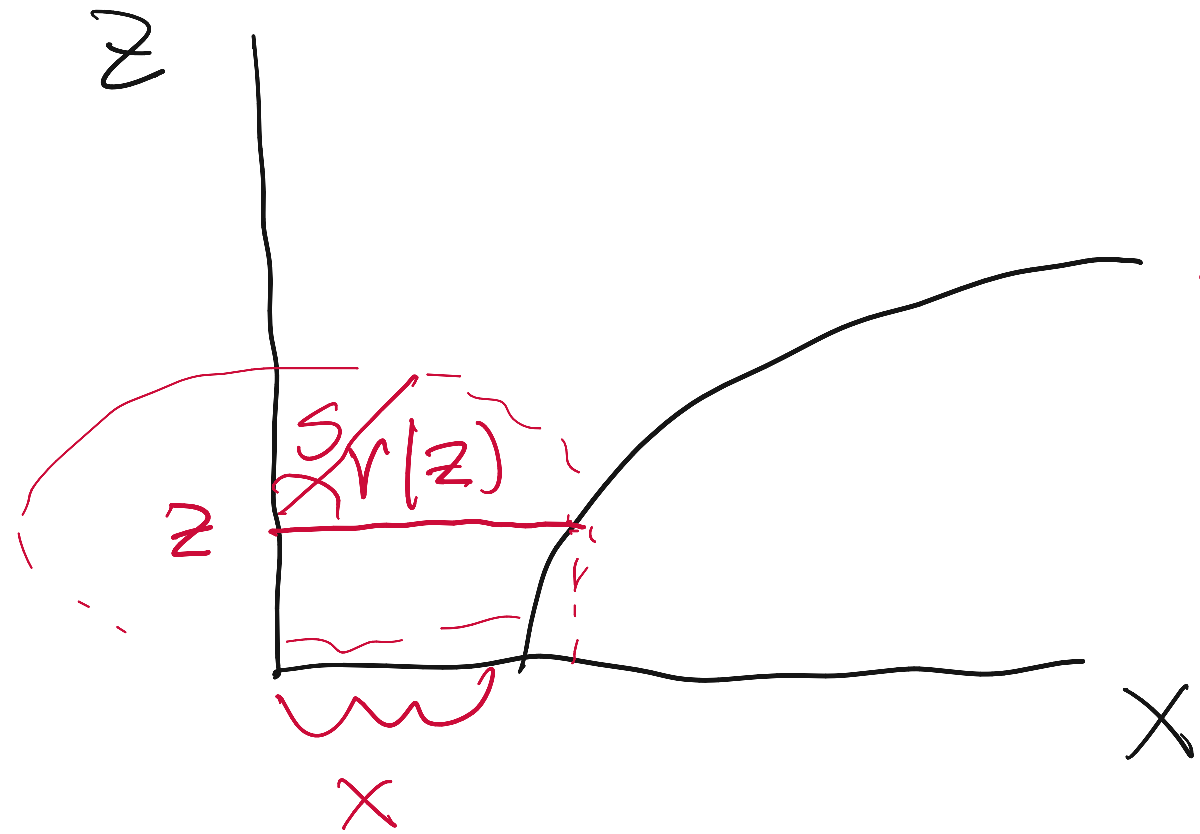
"Std trick"

$$\vec{r}(s,t) = \langle s, t, f(s,t) \rangle$$

$$\vec{r}(r,t) = \langle s, f(s,t), t \rangle \qquad y = f(x,z)$$

$$r(s,t) = \langle f(s,t), s, t \rangle \qquad x = f(y,z)$$

S is a "Surface of revolution"!



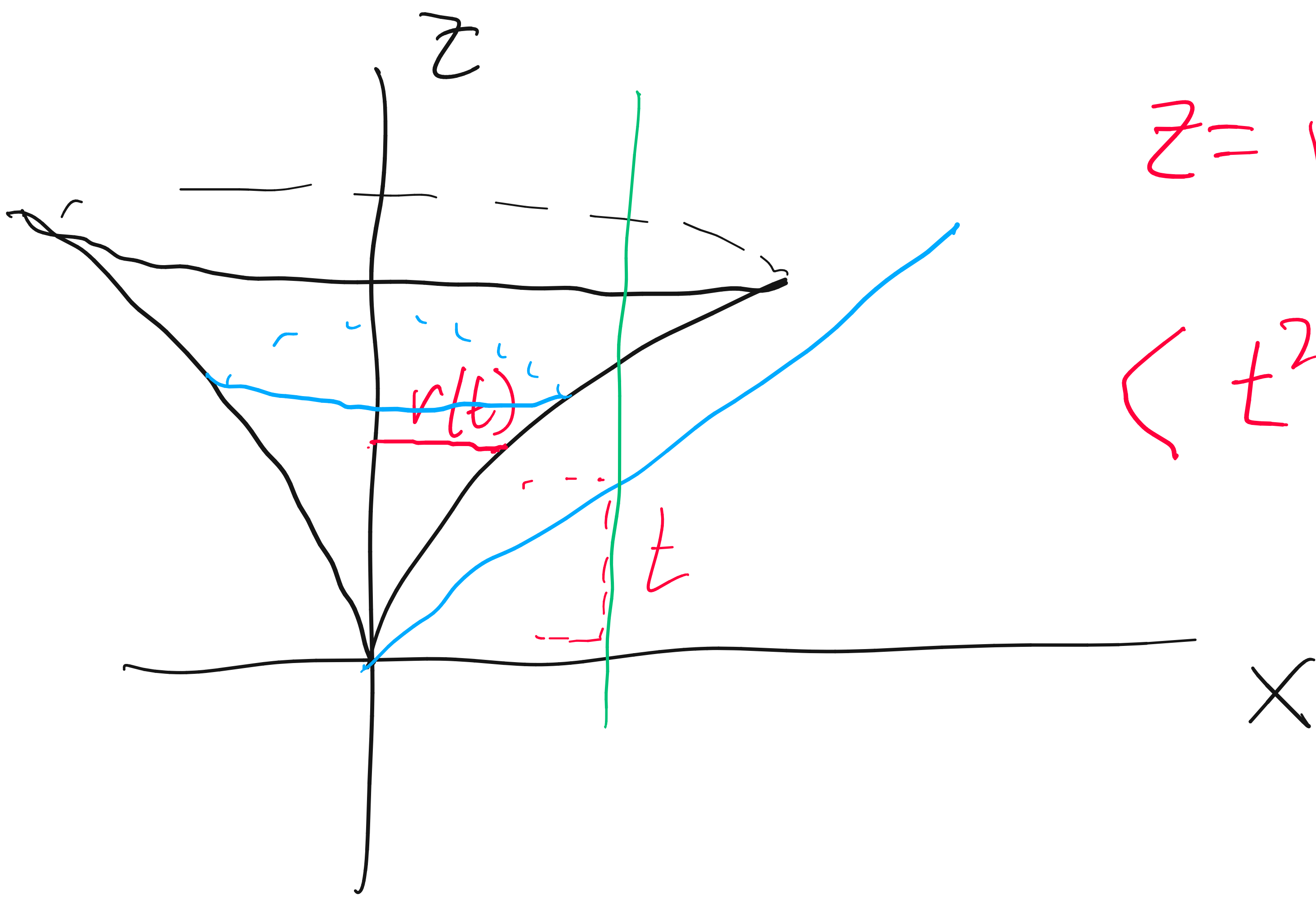
$z = f(x)$

$\Downarrow$

$x = \boxed{g(z) = r(z)}$ $z = t$

$$r(s,t) = \boxed{\langle r(t)\cos(s),\ r(t)\sin(s),\ t \rangle}$$

$z = \sqrt{x}$ about z-axis.



$z = \sqrt{x} \Rightarrow x = z^2$

$\langle t^2\cos(s), t^2\sin(s), t \rangle$

@ height t, radius $= t^2$.

$$\text{flux} = \iint_S \vec{F} \cdot \vec{n}\, dA = \iint \vec{F}(r(s,t)) \cdot (\vec{r}_s \times \vec{r}_t)\, dA$$